Quantum contextuality: A historical perspective

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QUANTERA ERA-NET Cofund in Quantum Technologies

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SECuRe quantum communication based on Energy-Time/time-bin entanglement

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QDISC Quantum Device-Independent Secure Communication

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Aim of this talk

- Historical introduction to quantum contextuality
- Differences between the Kochen-Specker theorem and the theory-independent notion of contextuality for ideal measurements
- Why contextuality for ideal measurements is key for understanding QT
- <u>Disclaimer</u>: I assume that the audience is familiar some of the results reviewed in the talk. I have no time to discuss details. My aim is to point out connections that may help us to understand the evolution of the field. The selections and emphases are subjective

1926-1927: The problem of "hidden variables"

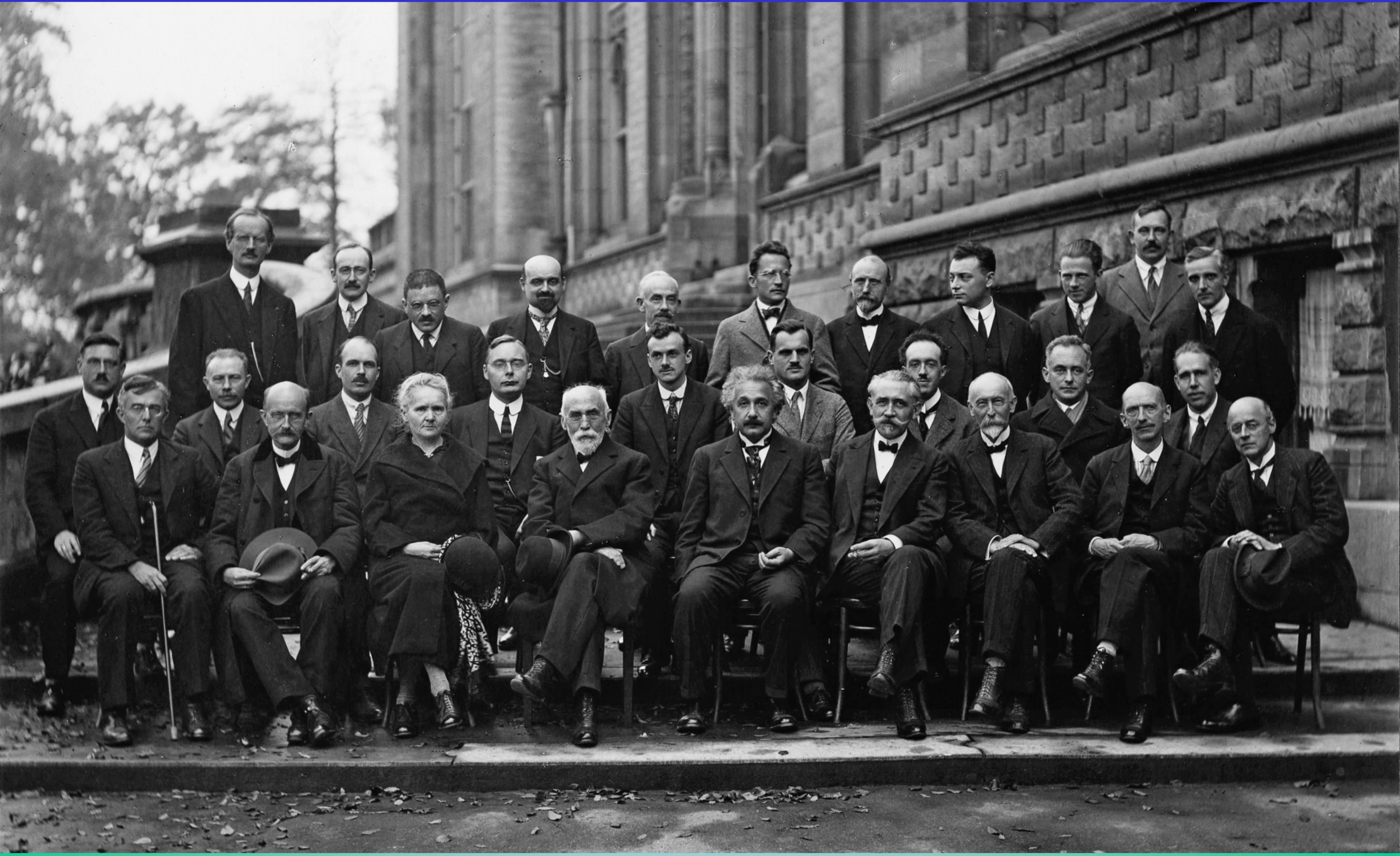
- Origin of the problem:
  - **Born's probabilistic interpretation** of Schrödinger's wave function, expressing the fundamentally probabilistic character of the predictions of quantum mechanics
  - **Heisenberg's uncertainty principle**, asserting a fundamental limit to the precision with which the values of position and momentum can be predicted in quantum mechanics

Born, M. (1926a), Z. Physik **38**, 803.
Born, M. (1926b), Z. Physik **37**, 863.
Heisenberg, W. (1927), Z. für Physik **43**, 172.

1926-1927: The problem of “hidden variables”

- Two sides:
  - **Heisenberg**, **Born**, **Pauli**, and **Bohr** made strong claims that quantum mechanics provided a complete framework for physics and manifested their skepticism about the possibility of completing it with hidden variables
  - **Schrödinger**, **de Broglie**, and **Einstein** hoped that incompatible observables such as position and momentum could be shown to have simultaneous values in a deeper non-probabilistic theory, and viewed the quantum state as an incomplete description in need of supplementation by hidden variables

1927: The Solvay conference



1927: The Solvay conference

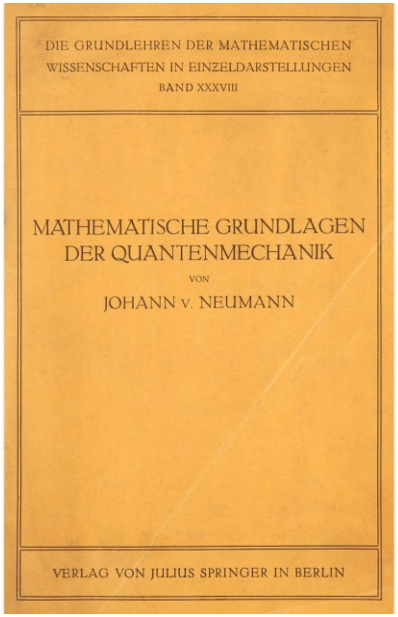
1927: The Solvay conference

- De Broglie presented an explicit hidden variable theory
- The criticisms received, particularly from Pauli, persuaded him to abandon his theory
- Recommended (film of the Solvay conference): https://www.youtube.com/watch?v=UK1LA6jlcgM

Lorentz, H. A. (1928), *Électrons et Photons—Rapports et Discussions du Cinquième Conseil de Physique tenu à Bruxelles du 24 au 29 Octobre 1927 sous les Auspices de L'Institut International de Physique Solvay* (Gauthier-Villars).

1931-1932: von Neuman's no-hidden variables proof

- Proof of impossibility of hidden variables
- Influential but later shown to be inconclusive



von Neumann, J. (1931), Ann. of Math. **32**, 191.
von Neumann, J. (1932), *Mathematische Grundlagen der Quantenmechanik* (Springer-Verlag, Berlin).

1932: Wigner's quasiprobability distribution

- When attempting to link Schrödinger's wave function to a distribution on phase space, Wigner found that such a distribution has negative values and cannot be made non-negative
- The importance of this was not recognized until much later

Wigner, E. (1932), Phys. Rev. **40**, 749.

1935: The EPR paper

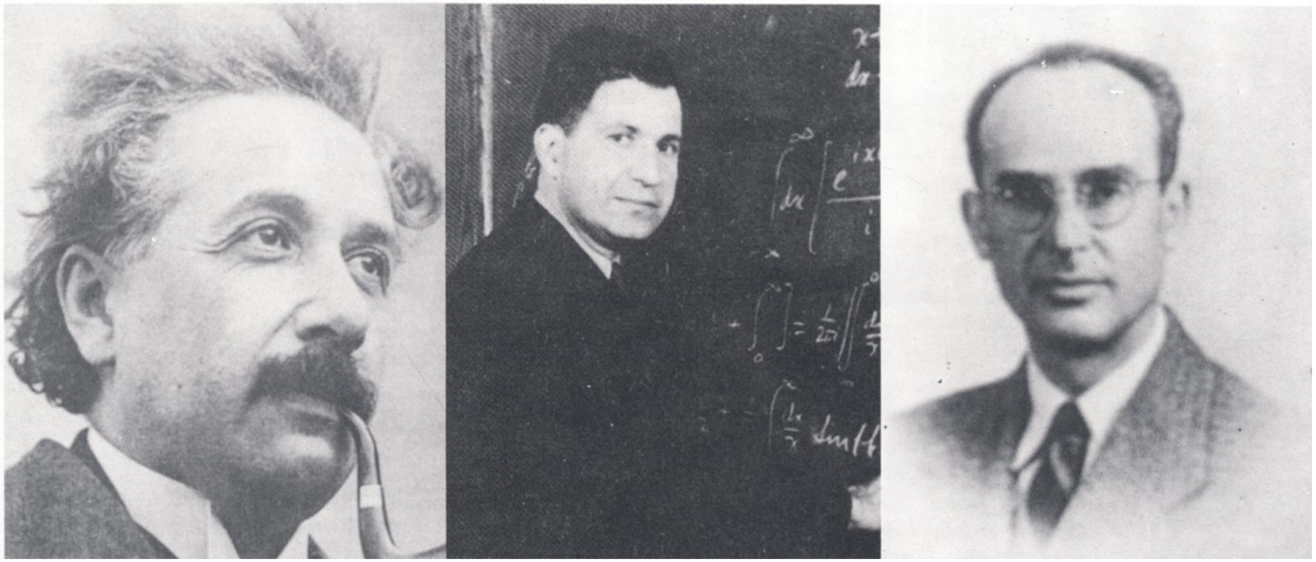
MAY 15, 1935 PHYSICAL REVIEW VOLUME 47

Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?

A. EINSTEIN, B. PODOLSKY AND N. ROSEN, *Institute for Advanced Study, Princeton, New Jersey*

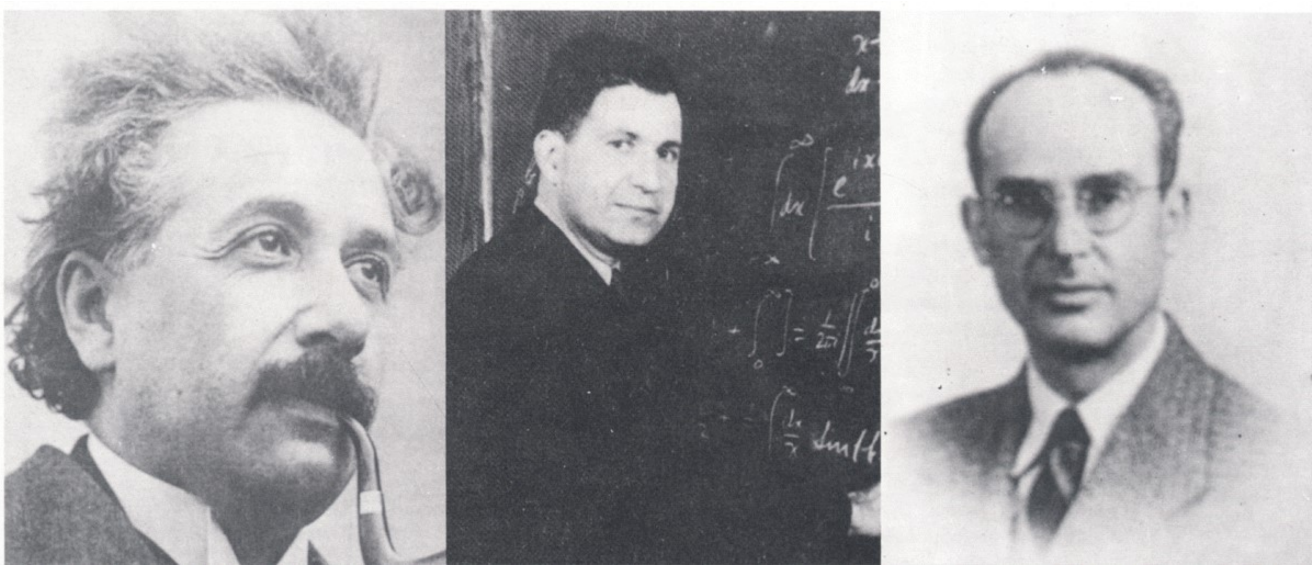
(Received March 25, 1935)

In a complete theory there is an element corresponding to each element of reality. A sufficient condition for the reality of a physical quantity is the possibility of predicting it with certainty, without disturbing the system. In quantum mechanics in the case of two physical quantities described by non-commuting operators, the knowledge of one precludes the knowledge of the other. Then either (1) the description of reality given by the wave function in quantum mechanics is not complete or (2) these two quantities cannot have simultaneous reality. Consideration of the problem of making predictions concerning a system on the basis of measurements made on another system that had previously interacted with it leads to the result that if (1) is false then (2) is also false. One is thus led to conclude that the description of reality as given by a wave function is not complete.



1935: The EPR paper

- Quantum mechanics is incomplete, in the sense that it does not assign definite outcomes to measurements whose results can be predicted with certainty from the outcomes of space-like separated measurements
- Bell will show that EPR's hidden variable theories collide with quantum mechanics but, at that time, the EPR argument reinforced Einstein's resistance to accept quantum mechanics as a final theory



1936: Quantum logic

- Birkhoff and von Neumann developed a quantum logic, a set of algebraic rules governing operations to combine and predicates to relate propositions associated with physical events

- This logic will provide a new basis for discussing the problem of hidden variables

Birkhoff, G., and J. von Neumann (1936), Ann. Math. **37**, 823.

1952: Bohm’s hidden variables theory

- Further elaboration of de Broglie’s theory of 1927
- Deterministic and explicitly non-local at the level of hidden variables

PHYSICAL REVIEW VOLUME 85, NUMBER 2 JANUARY 15, 1952

A Suggested Interpretation of the Quantum Theory in Terms of “Hidden” Variables. I

DAVID BOHM*
Palmer Physical Laboratory, Princeton University, Princeton, New Jersey

1957: A generalized probability theory for QM

- Mackey asked whether every measure on the lattice of projections of a Hilbert space can be defined by a positive operator with unit trace

- A positive answer would show that the Born rule follows from a particular set of axioms (framing a generalized probability theory) for quantum mechanics

Mackey, G. W. (1957), Amer. Math. Monthly **64**, 45.

Mackey, G. W. (1963), *Mathematical Foundations of Quantum Mechanics* (Addison-Wesley).

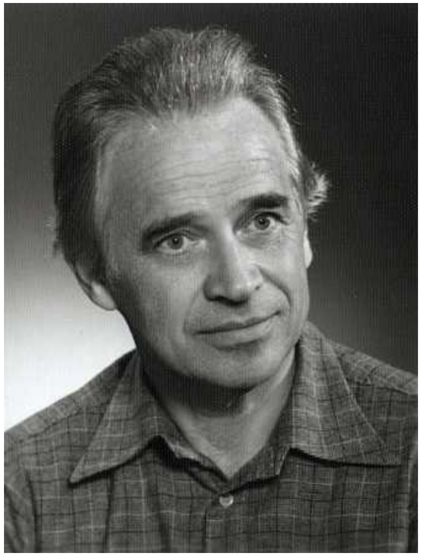
1957: Gleason’s theorem

- Kadison (and later Bell and Kochen and Specker) proved this false for two-dimensional Hilbert spaces
- Gleason showed it to be true for higher dimensions
- Gleason’s theorem is going to play a crucial role in the discussion of hidden variables

Gleason, A. M. (1957), J. Math. Mech. **6**, 885.

1960: Specker

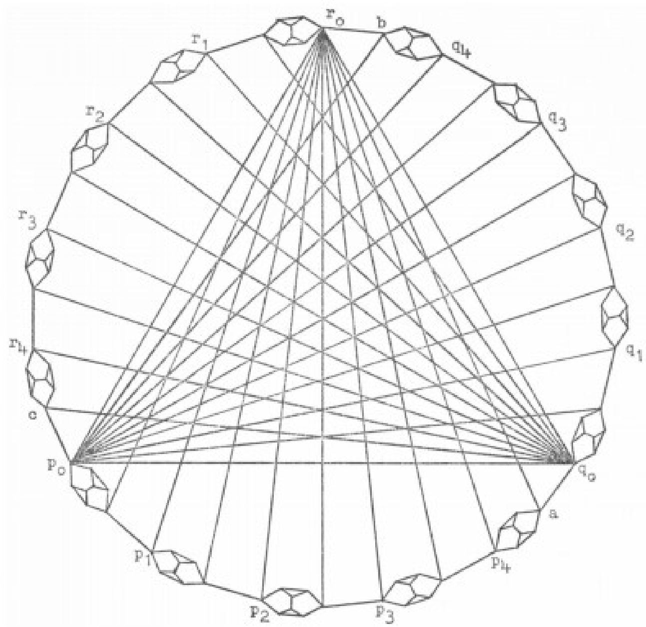
- Inspired by "the question whether the omniscience of God also extends to events that would have occurred in case something would have happened that did not happen" and by the logic of Birkhoff and von Neumann, Specker reformulated the question of hidden variables as follows: "Is it possible to extend the description of a quantum mechanical system through the introduction of supplementary --fictitious-- propositions in such a way that in the extended domain the classical propositional logic holds?"



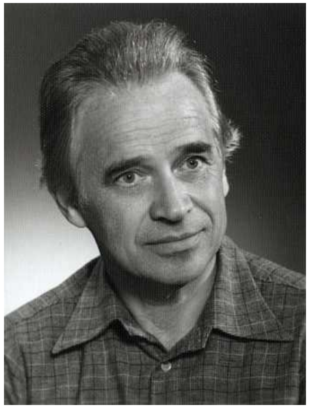
- Specker found that "[t]he answer to this question is negative, except in the case of Hilbert spaces of dimension 1 and 2" as "[a]n elementary geometrical argument shows"

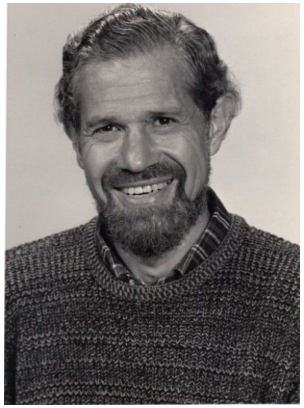
Specker, E. (1960), Dialectica **14** (2–3), 239.

1967: The Kochen-Specker theorem



E. P. Specker, Adrian Specker, and S. Kochen at Rigiblick, Zürich, January 1963. Courtesy of Suzanne Specker



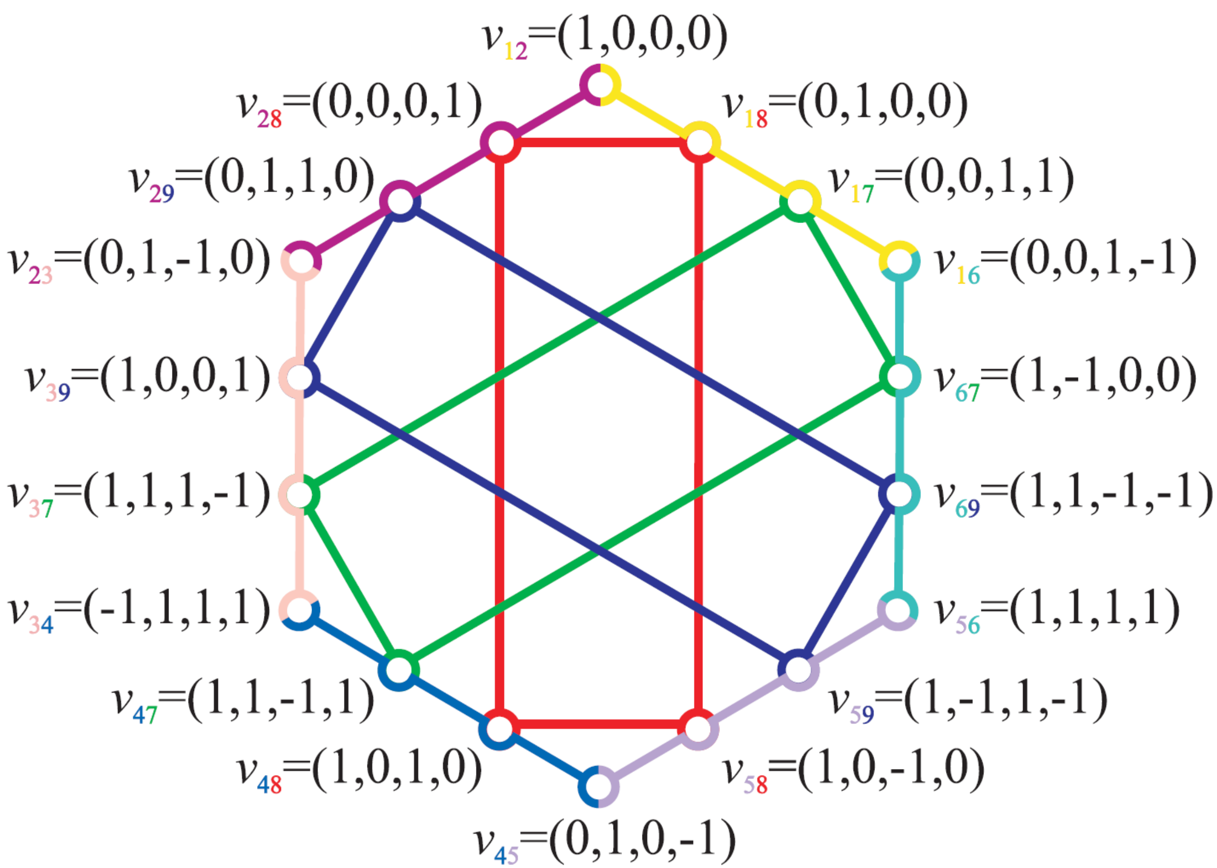


Kochen, S., and E. P. Specker (1965a), in *Proceedings of the 1964 International Congress for Logic, Methodology and Philosophy of Science, Jerusalem*, edited by Y. Bar-Hillel (North-Holland, Amsterdam) pp. 45–57.

Kochen, S., and E. P. Specker (1965b), in *Symposium on the Theory of Models: Proceedings of the 1963 International Symposium at Berkeley*, edited by J. W. Addison, L. Henkin, and A. Tarski (North-Holland, Amsterdam) pp. 177–189.

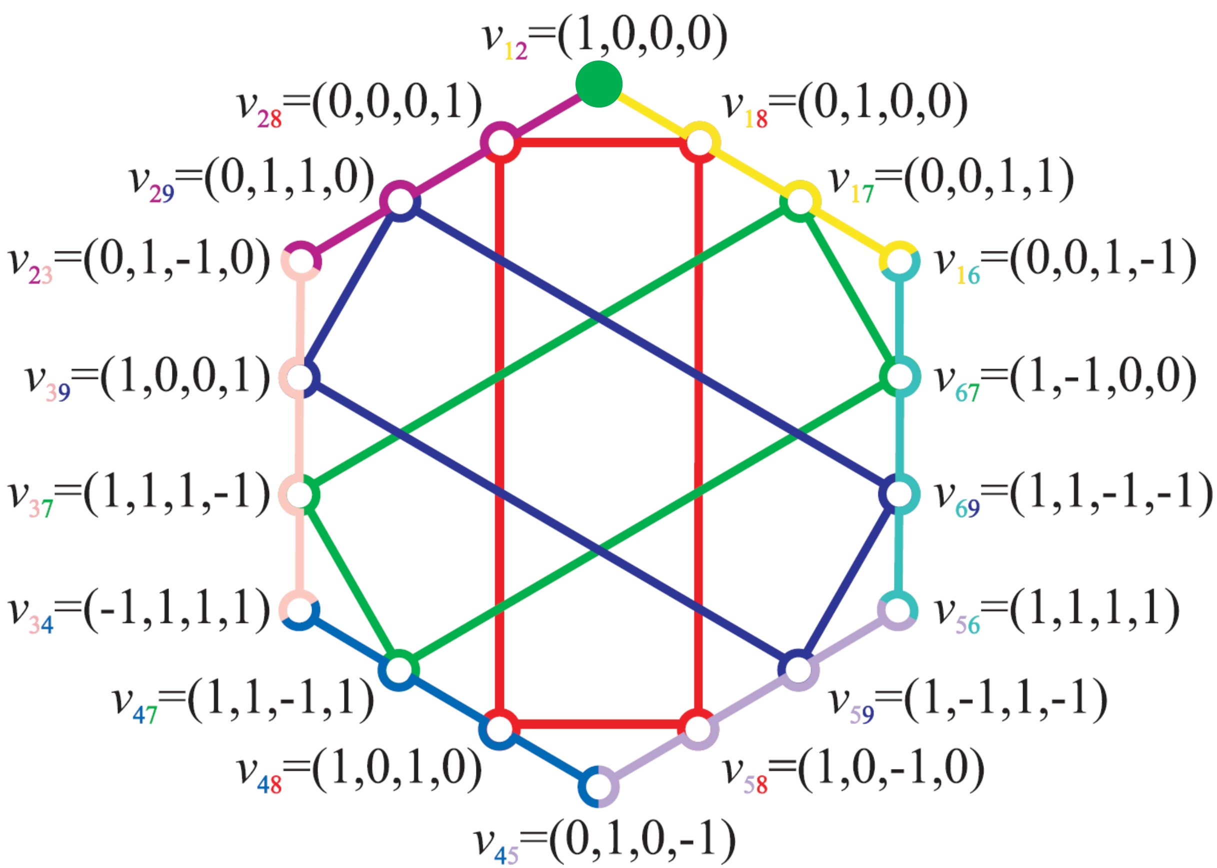
Kochen, S., and E. P. Specker (1967), J. Math. Mech. **17** (1), 59.

The 18-vector proof of the KS theorem



A. Cabello, J. M. Estebaranz, and G. García-Alcaine, Phys. Lett. A **212**, 183 (1996).

The 18-vector proof of the KS theorem



A. Cabello, J. M. Estebaranz, and G. García-Alcaine, Phys. Lett. A **212**, 183 (1996).

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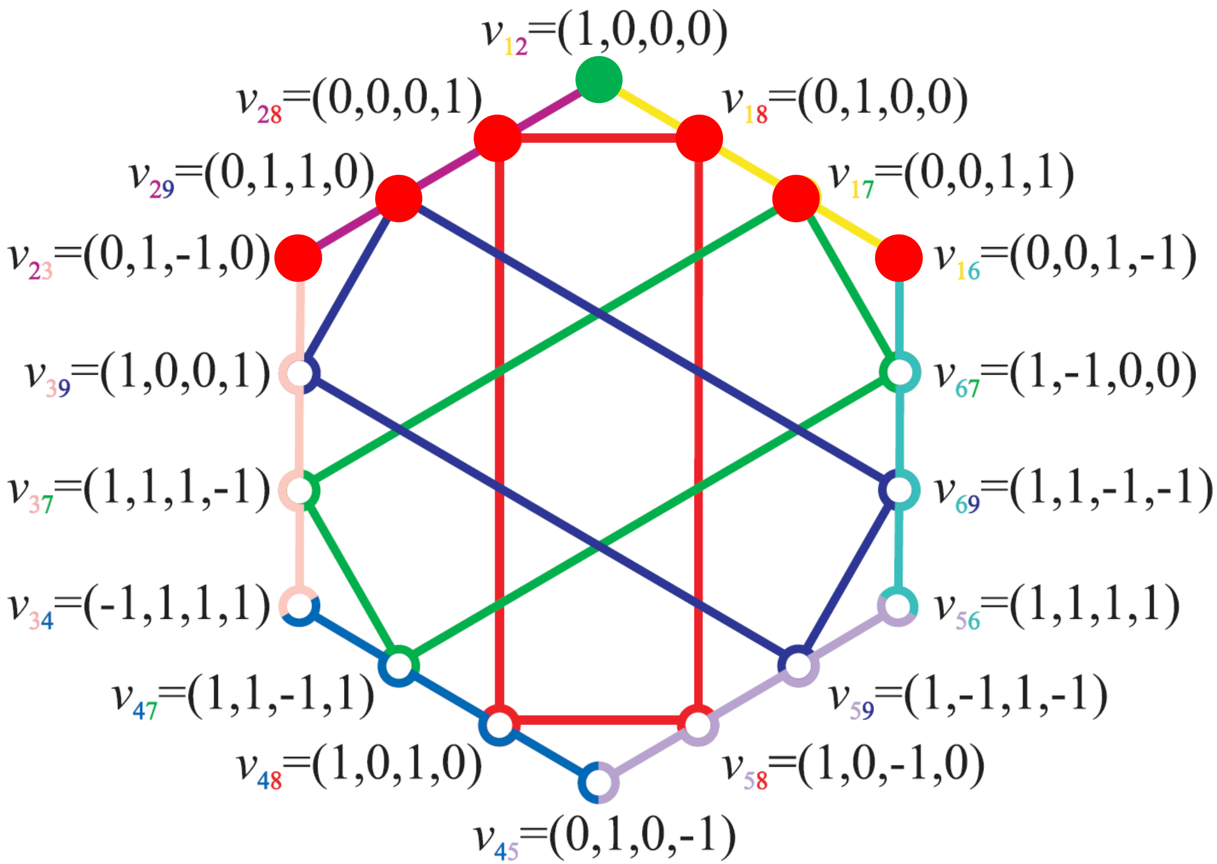

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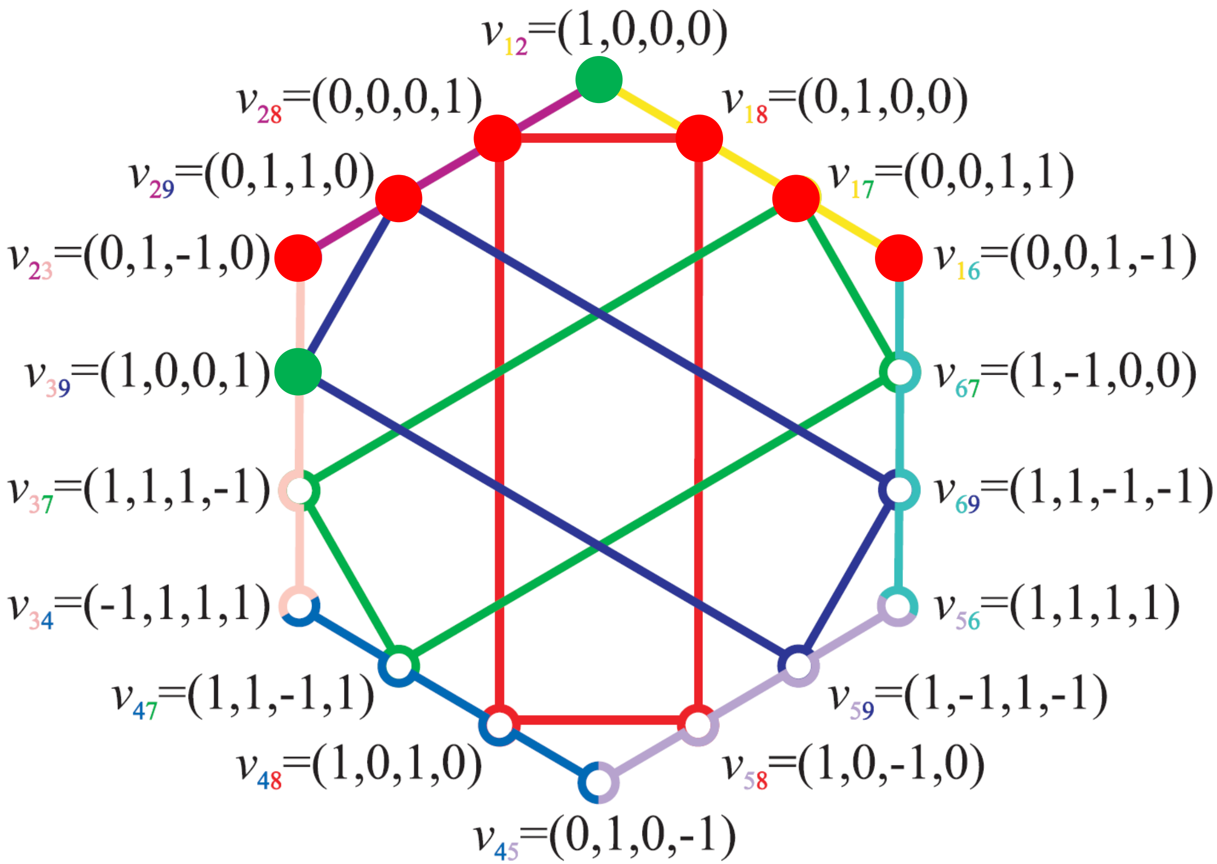

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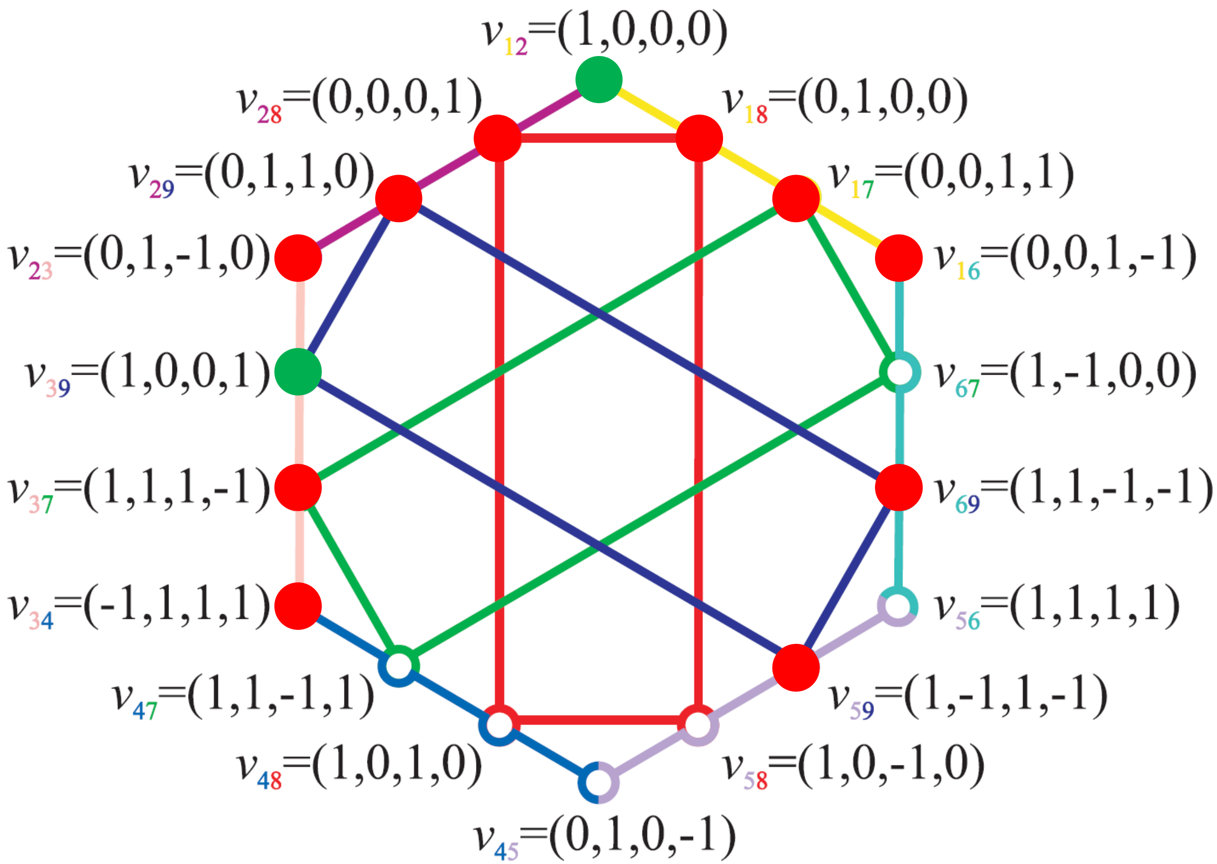

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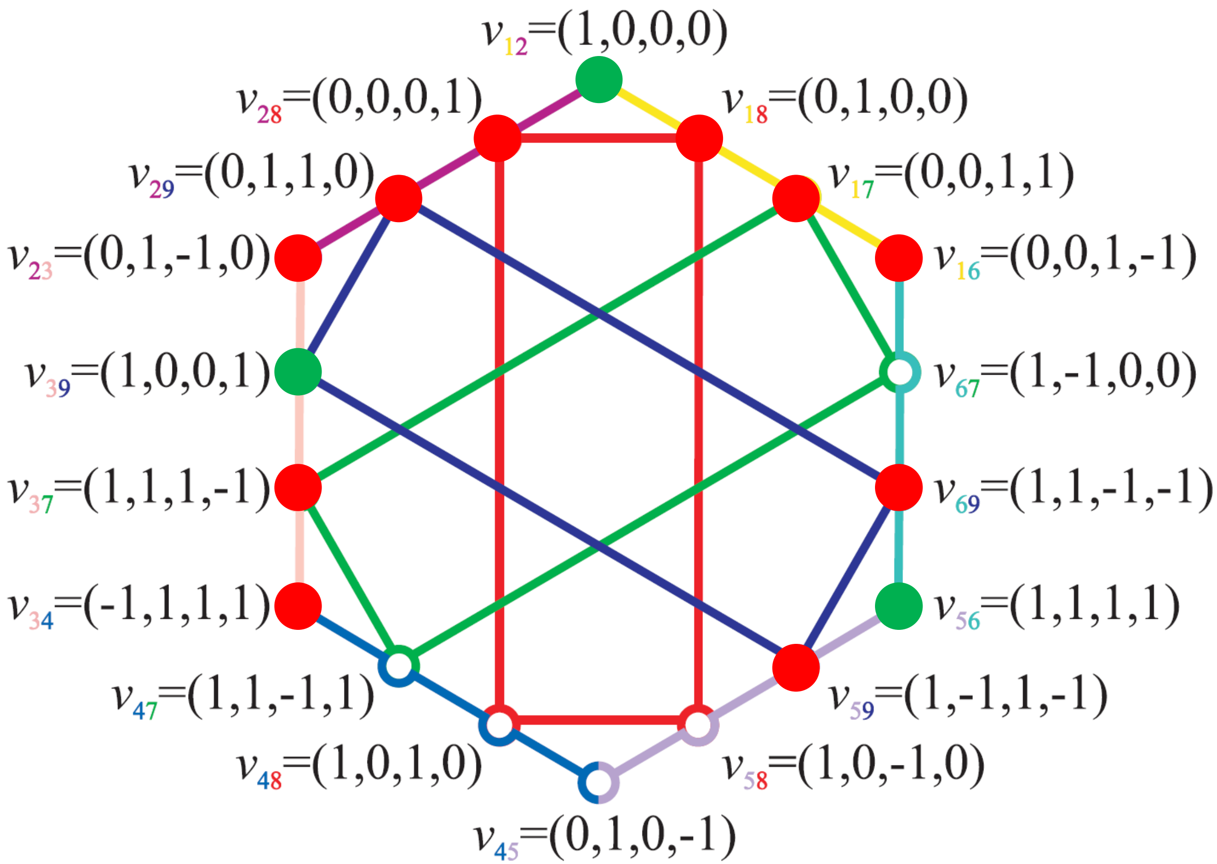

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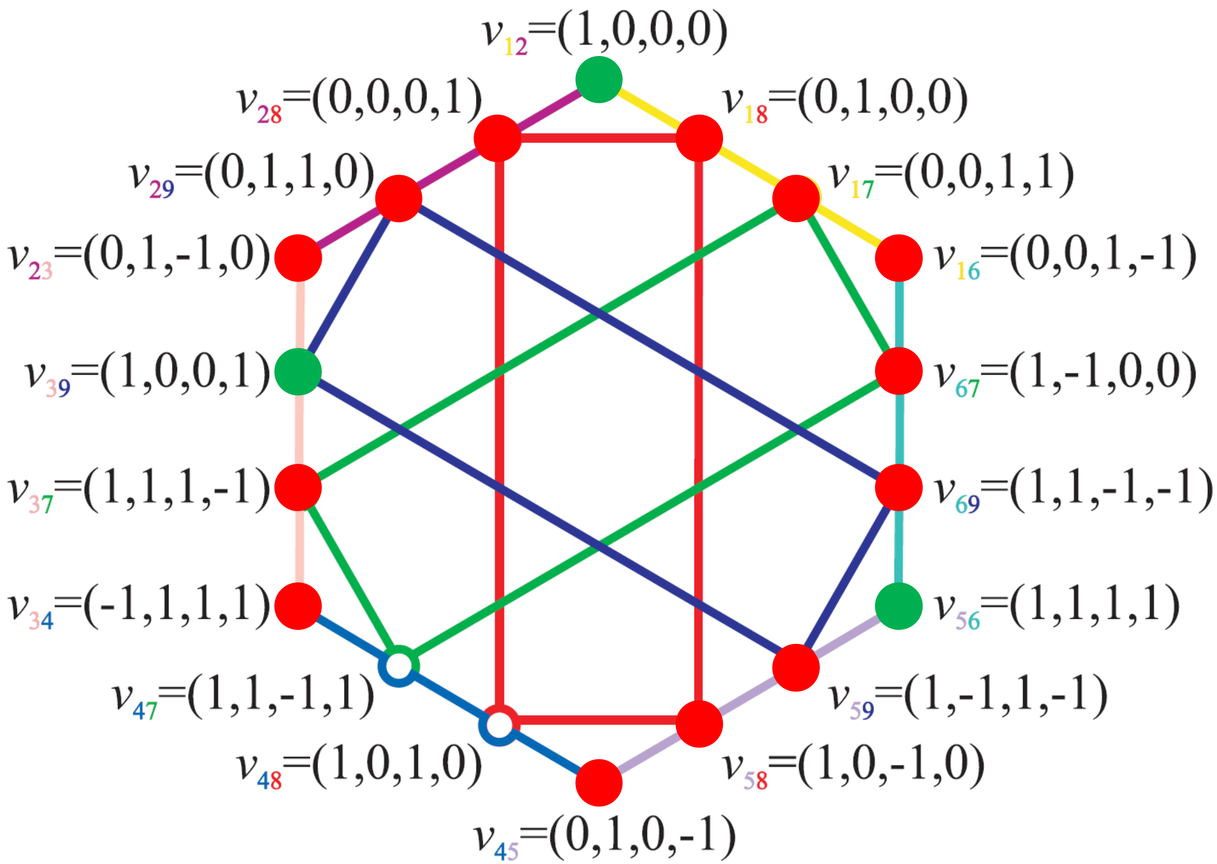

A. Cabello, J.M. Estebaranz, and G. García-Alcaine, Phys. Lett. A **212**, 183 (1996).

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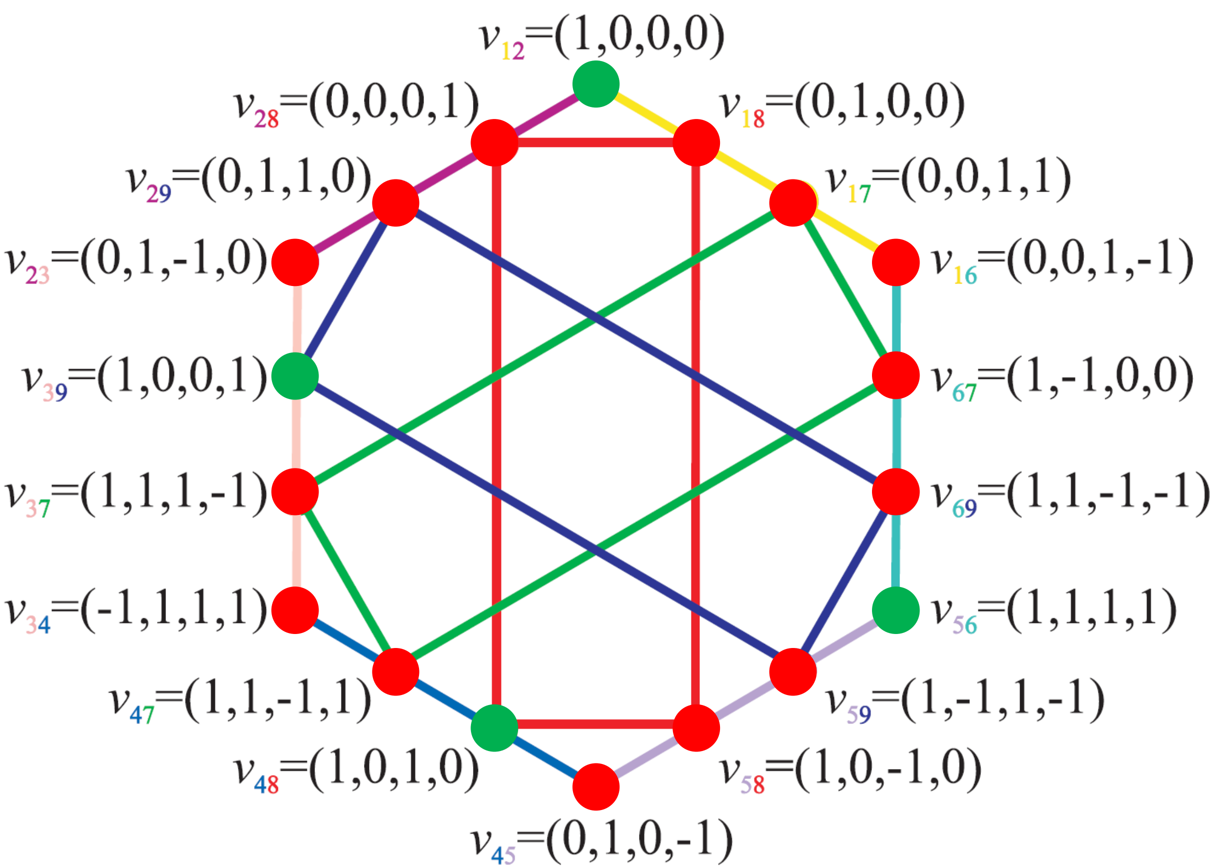

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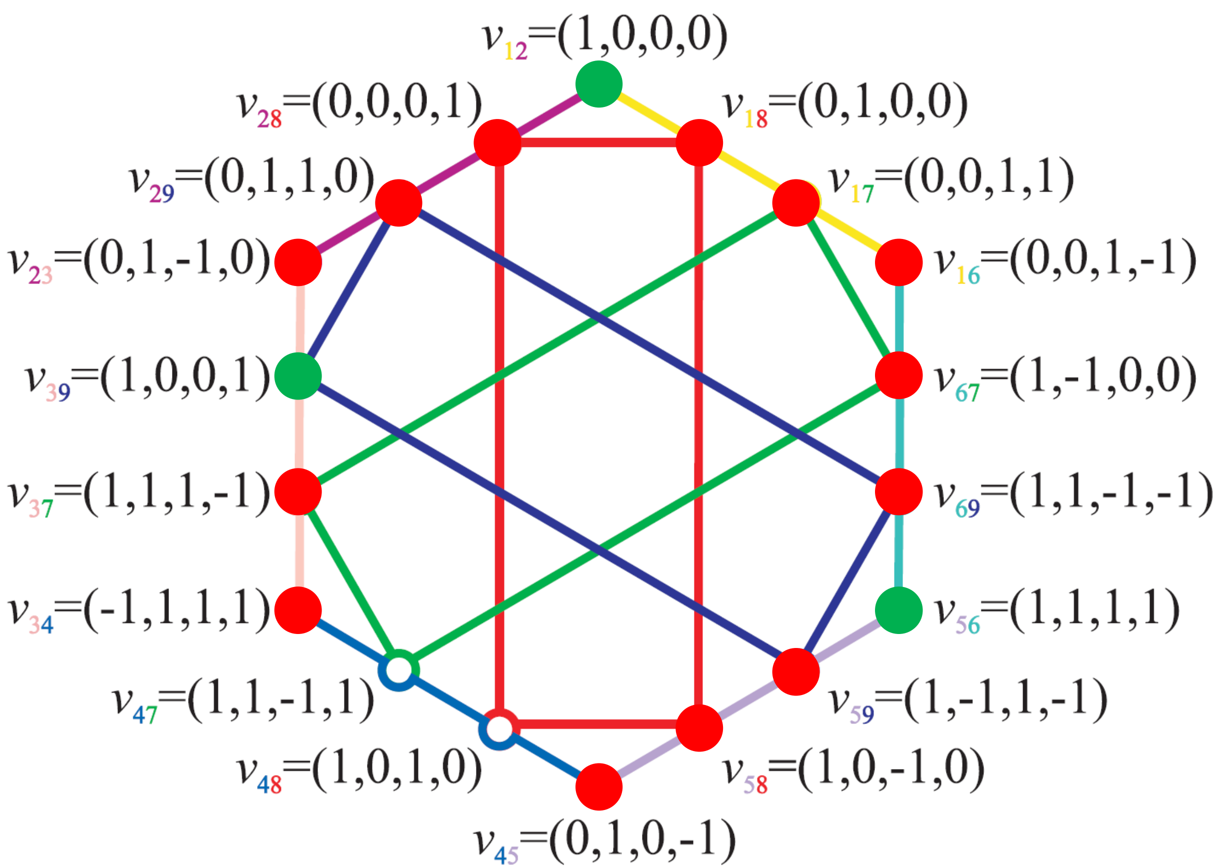

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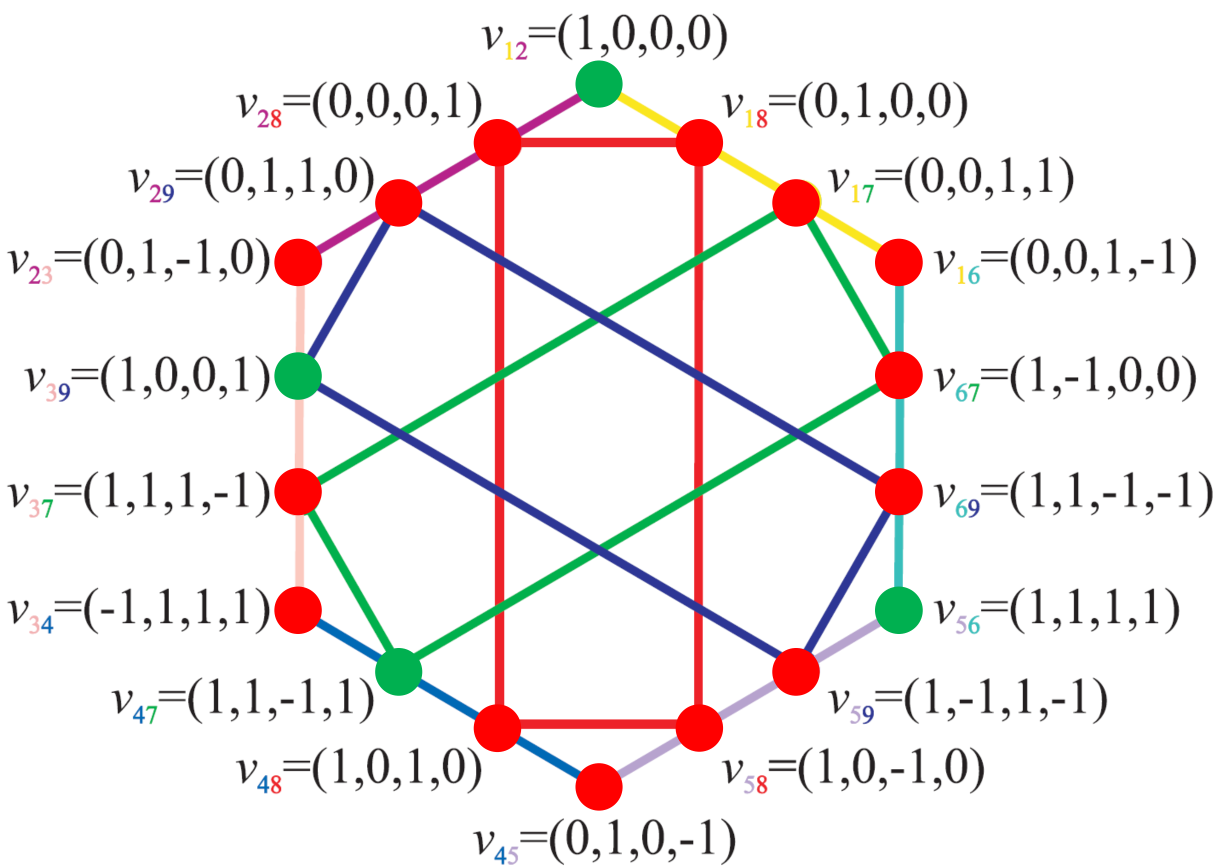
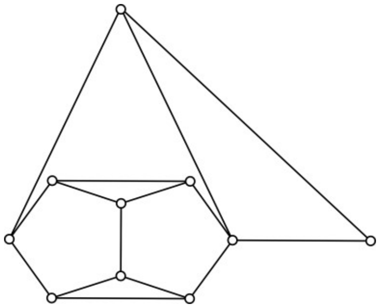


A. Cabello, J.M. Estebaranz, and G. García-Alcaine, Phys. Lett. A **212**, 183 (1996).

1963-1966: Bell's NCHV proof

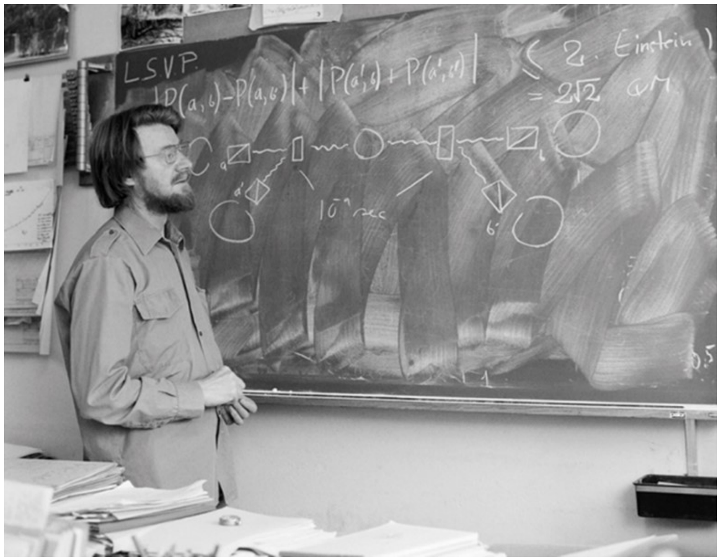
- More complex basic structure than KS's

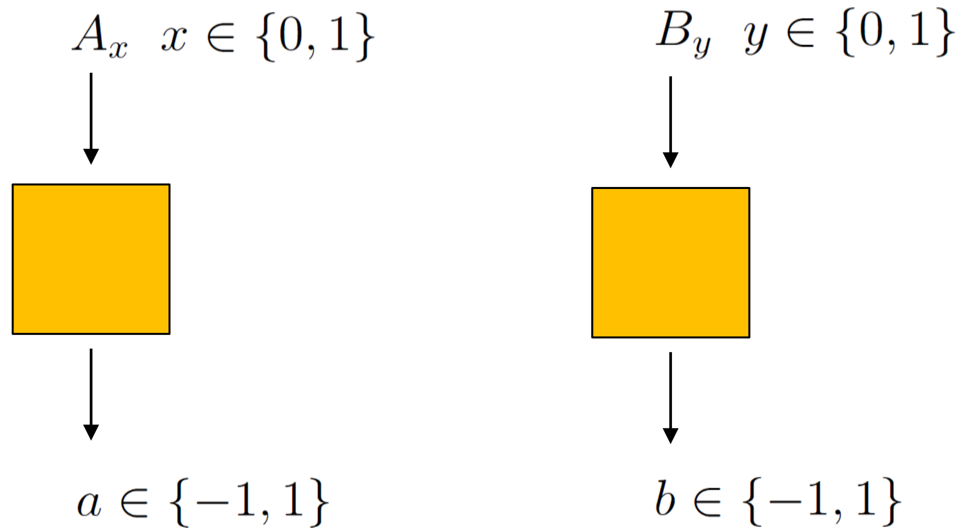


- Infinite observables
- Bell is skeptical about the assumption of outcome non-contextuality (measurements in different contexts require different devices)

Bell, J. S. (1966), Rev. Mod. Phys. **38**, 447.

1964: Bell's theorem and inequality

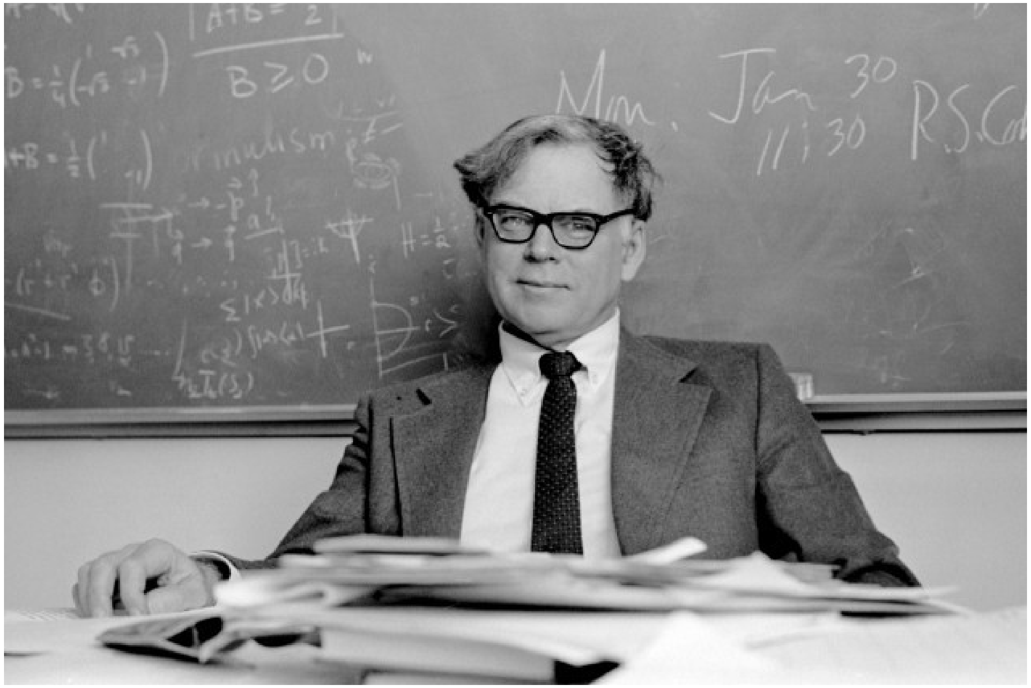




$$\langle A_0 B_0 \rangle + \langle A_0 B_1 \rangle + \langle A_1 B_0 \rangle - \langle A_1 B_1 \rangle \overset{\text{LR}}{\leq} 2 \overset{\text{QT}}{\leq} 2\sqrt{2}$$

J. Bell, Physics **1**, 195 (1964).
J. F. Clauser, M. A. Horne, A. Shimony, and R. A. Holt, Phys. Rev. Lett. **23**, 880 (1969).

1971-1981: The origin of the word "contextuality"



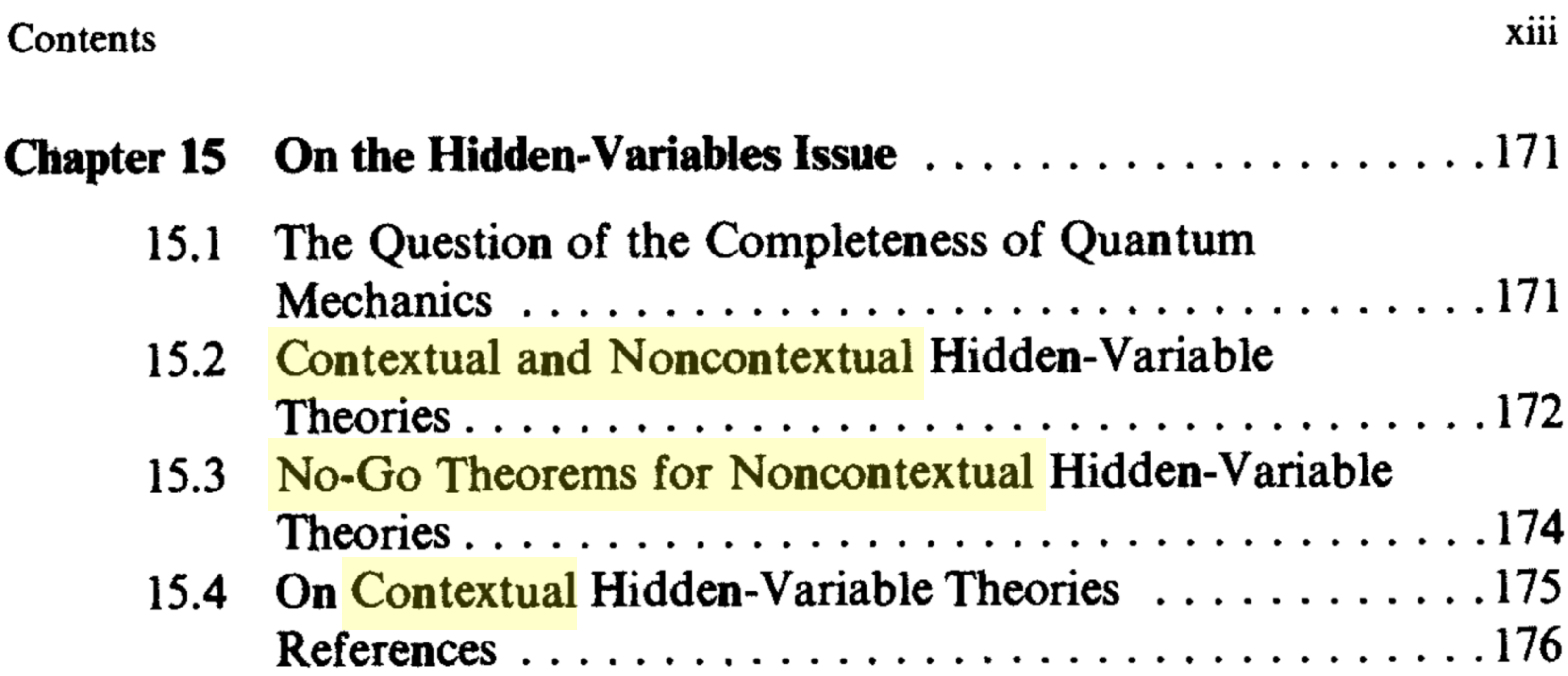
Shimony, A. (1971), in *Foundations of Quantum Mechanics: Proceedings of the International School of Physics "Enrico Fermi", Course IL, Varenna on Lake Como, Villa Monastero, 29th June–11th July 1970*, edited by B. D'Espagnat (Academic Press, New York) pp. 182–194.

Jaeger, G. (2019), Phil. Trans. R. Soc. A **377**, 20190025.

1971: “Contextualistic” hidden-variable theories

The study of hidden-variable interpretations of quantum mechanics was transformed by two papers of Bell. In his 1966 paper in *Reviews of Modern Physics* [1] he analysed the argument of von Neumann [2] and the more powerful arguments of Jauch and Piron [3] and of Gleason [4], all leading to similar conclusions about the nonassignability of simultaneous values to all the observables of a nontrivial quantum system; and he showed that these arguments do not exclude what may be called “contextualistic” hidden-variable theories, in which the value of an observable O is allowed to depend not only upon the hidden state λ, but also upon the set C of compatible observables measured along with O. In Bell’s 1964 paper in *Physics* [5] (written later than the other, in spite of the earlier publication date) he gave a new kind of argument against a hidden-variable interpretation of quantum mechanics, by showing that no hidden-variables theory which satisfies a certain condition of locality can agree with all the statistical predictions of quantum mechanics.

1981: “Contextual” hidden-variable theories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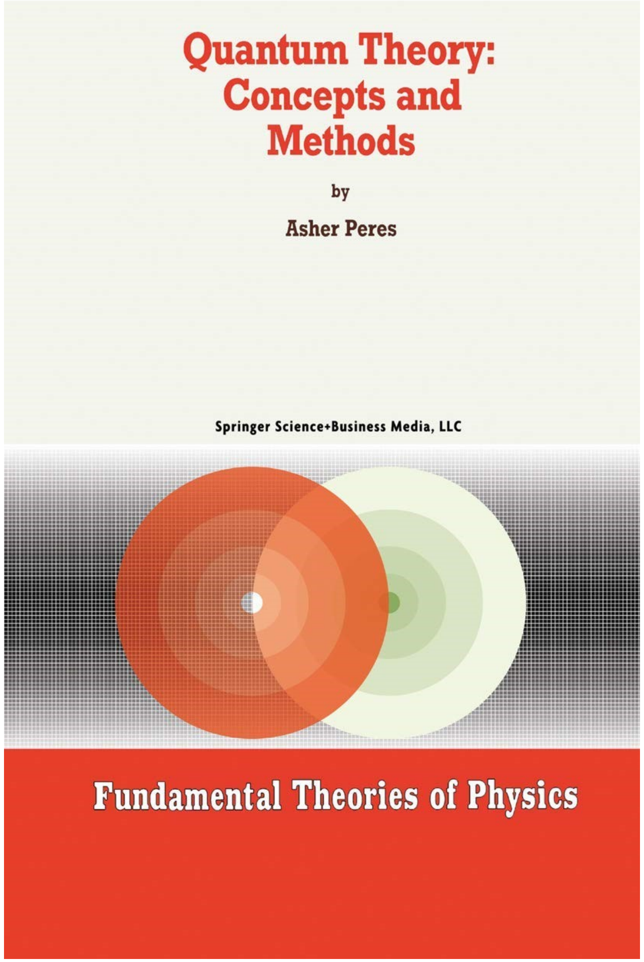
1993: "Contextuality"

Chapter 7: Contextuality

7-1. Nonlocality versus contextuality
7-2. Gleason's theorem
7-3. The Kochen-Specker theorem
7-4. Experimental and logical aspects of contextuality
7-5. Appendix: Computer test for Kochen-Specker contradiction
7-6. Bibliography



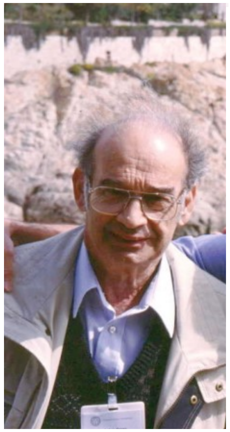
Peres, A. (1993), *Quantum Theory: Concepts and Methods* (Kluwer, Dordrecht).

1990: The Peres-Mermin square

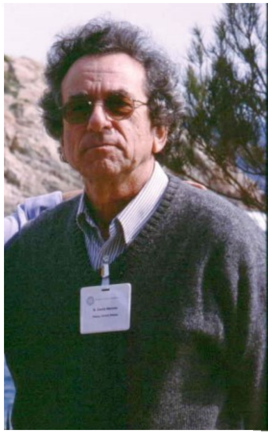
$$
\begin{array}{ccc}
\sigma_z^{(1)} & \sigma_z^{(2)} & \sigma_z^{(1)} \otimes \sigma_z^{(2)} \\
\sigma_x^{(2)} & \sigma_x^{(1)} & \sigma_x^{(1)} \otimes \sigma_x^{(2)} \\
\sigma_z^{(1)} \otimes \sigma_x^{(2)} & \sigma_x^{(1)} \otimes \sigma_z^{(2)} & \sigma_y^{(1)} \otimes \sigma_y^{(2)}
\end{array}
$$

Greenberger, Horne, Zeilinger

Peres



Mermin



Peres, A. (1990a), Phys. Lett. A **151** (3–4), 107.

Peres, A. (1992), Found. Phys. **22**, 357.

Mermin, N. D. (1990c), Phys. Rev. Lett. **65** (27), 3373.

Mermin, N. D. (1993), Rev. Mod. Phys. **65**, 803.

1990: Mermin's unification

- GHZ can be converted into a Bell inequality

VOLUME 65, NUMBER 15 PHYSICAL REVIEW LETTERS 8 OCTOBER 1990

Extreme Quantum Entanglement in a Superposition of Macroscopically Distinct States

N. David Mermin

Laboratory of Atomic and Solid State Physics, Cornell University, Ithaca, New York 14853-2501

(Received 29 May 1990)

- GHZ can be extended into a KS proof

VOLUME 65, NUMBER 27 PHYSICAL REVIEW LETTERS 31 DECEMBER 1990

Simple Unified Form for the Major No-Hidden-Variables Theorems

N. David Mermin

Laboratory of Atomic and Solid State Physics, Cornell University, Ithaca, New York 14853-2501

(Received 9 October 1990)

1983: Kochen-Specker with locality

Allen Stairs in the 80's

Michael Redhead

Kochen, S. (????), "letter to A. Shimony, reported in (Stairs, 1983) and in (Heywood and Redhead, 1983a)," .

Stairs, A. (1983), Philos. Sci. **50**, 578.

Heywood, P., and M. L. G. Redhead (1983b), Found. Phys. **13** (5), 481.

Unifying Bell and KS theorems?

- Bell's theorem leads to experimental tests of whether the world can be explained with theories which can be defined without any reference to quantum mechanics. In them, any observable is measured with the same device in every context

- The KS theorem is attached to quantum mechanics:

  - The KS theorem does not refer to general measurements, but to those that are represented in quantum mechanics by self-adjoint operators. (There are other measurements in quantum mechanics)

  - The proof of the KS theorem includes constraints that are specific to quantum systems. (E.g., KS, PM)

  - The experimental translation of the KS theorem (as proposed by KS and Bell) assumes quantum mechanics, as it is assumed that coarse-grainings of two different (and incompatible) measurements represent the same observable based on the fact that, in quantum mechanics, both yield the same outcome statistics

1998: Towards experimental falsification of NCHVTs

"the whole notion of an experimental test of [B]KS misses the point"

Mermin

Cabello, A., and G. García-Alcaine (1998), Phys. Rev. Lett. **80**, 1797.

Michler, M., H. Weinfurter, and M. Zukowski (2000), Phys. Rev. Lett. **84**, 5457.

Hasegawa, Y., R. Loidl, G. Badurek, M. Baron, and H. Rauch (2003), Nature (London) **425**, 45.

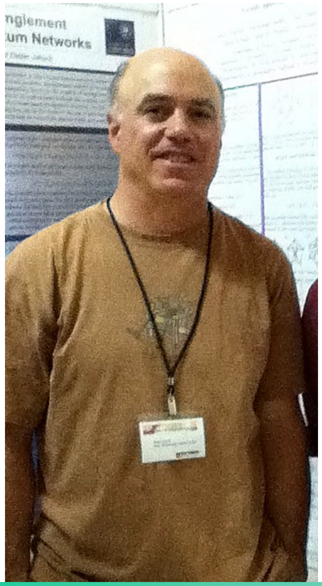
1999: Finite precision nullifies the KS theorem?

PHYSICAL REVIEW LETTERS

VOLUME 83 | 8 NOVEMBER 1999 | NUMBER 19

Finite Precision Measurement Nullifies the Kochen-Specker Theorem

David A. Meyer*



Meyer, D. A. (1999), Phys. Rev. Lett. **83** (19), 3751.

Kent, A. (1999), Phys. Rev. Lett. **83**, 3755.

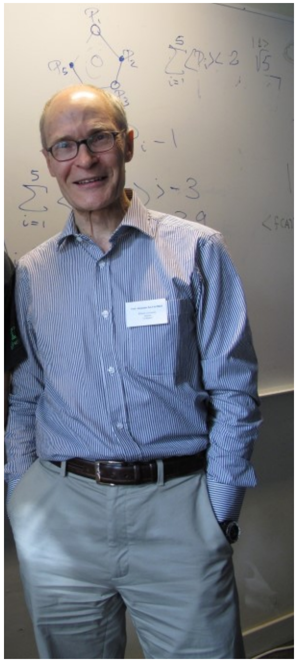
Clifton, R. K., and A. Kent (2000), Proc. R. Soc. Lond. A **456**, 2101.

2000: “KS inequalities” (still assuming QM)

Simon, C., i. c. v. Brukner, and A. Zeilinger (2001a), Phys. Rev. Lett. **86**, 4427.

Larsson, J.-A. (2002), Europhys. Lett. **58**, 799.

2008: The KCBS inequality



$$-\langle C_1C_2\rangle - \langle C_2C_3\rangle - \langle C_3C_4\rangle - \langle C_4C_5\rangle - \langle C_5C_1\rangle \overset{\text{NCHV}}{\leq} 3 \overset{\text{QT}}{\leq} 4\sqrt{5} - 5 \approx 3.944.$$

A. A. Klyachko, in *Physics and Theoretical Computer Science: From Numbers and Languages to (Quantum) Cryptography*, edited by J.-P. Gazeau, J. Nešetřil, and B. Rovan, NATO Security Through Science Series: Information and Communication Security vol. 7 (IOP Press, London, 2007), p. 25.

A. A. Klyachko, M. A. Can, S. Binicioğlu, and A. S. Shumovsky, Phys. Rev. Lett. **101**, 020403 (2008).

2008: The Peres-Mermin inequality

$$\langle ABC\rangle + \langle abc\rangle + \langle \alpha\beta\gamma\rangle + \langle Aa\alpha\rangle + \langle Bb\beta\rangle - \langle Cc\gamma\rangle \leq 4$$

- The inequality holds under the assumption of outcome non-contextuality
- Quantum mechanics is not assumed in any way

Cabello, A. (2008), Phys. Rev. Lett. **101** (21), 210401.

2008: The Peres-Mermin inequality

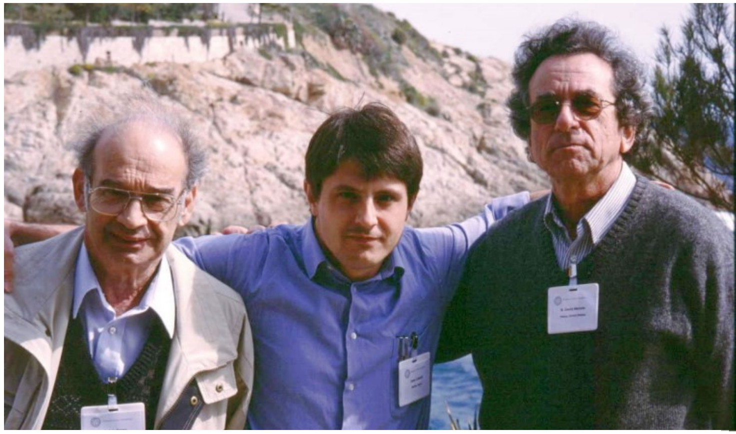
$$\langle ABC\rangle + \langle abc\rangle + \langle \alpha\beta\gamma\rangle + \langle Aa\alpha\rangle + \langle Bb\beta\rangle - \langle Cc\gamma\rangle \leq 4$$

$$A = \sigma_z^{(1)} \qquad B = \sigma_z^{(2)} \qquad C = \sigma_z^{(1)} \otimes \sigma_z^{(2)}$$

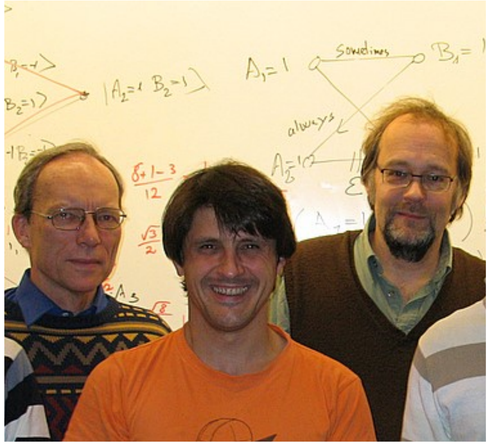
$$a = \sigma_x^{(2)} \qquad b = \sigma_x^{(1)} \qquad c = \sigma_x^{(1)} \otimes \sigma_x^{(2)}$$

$$\alpha = \sigma_z^{(1)} \otimes \sigma_x^{(2)} \qquad \beta = \sigma_x^{(1)} \otimes \sigma_z^{(2)} \qquad \gamma = \sigma_y^{(1)} \otimes \sigma_y^{(2)}$$

- The inequality holds under the assumption of outcome non-contextuality
- Quantum mechanics is not assumed in any way
- The quantum violation occurs for all 4-dim states using the Peres-Mermin observables



2009: Every KS set leads to a SI violation of a NCI



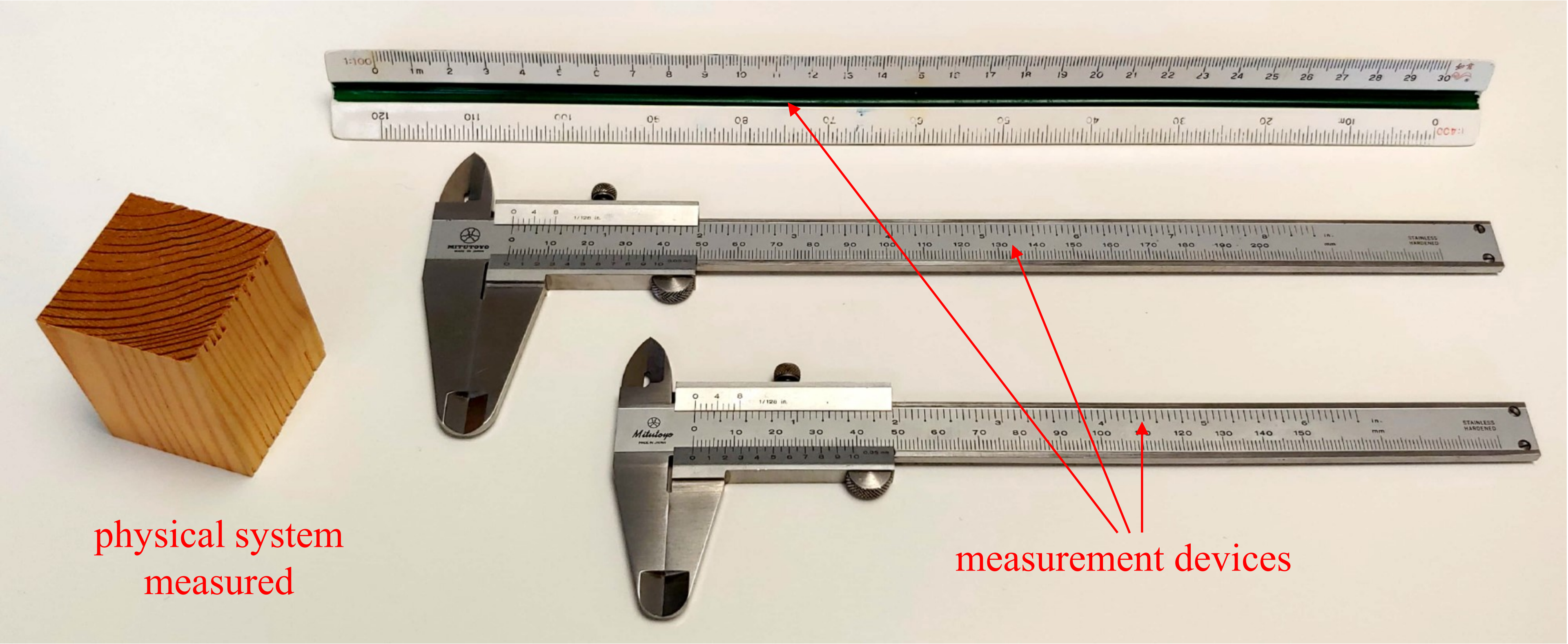
Badziag, P., I. Bengtsson, A. Cabello, and I. Pitowsky (2009), Phys. Rev. Lett. **103**, 050401.

Ideal (aka sharp) measurements

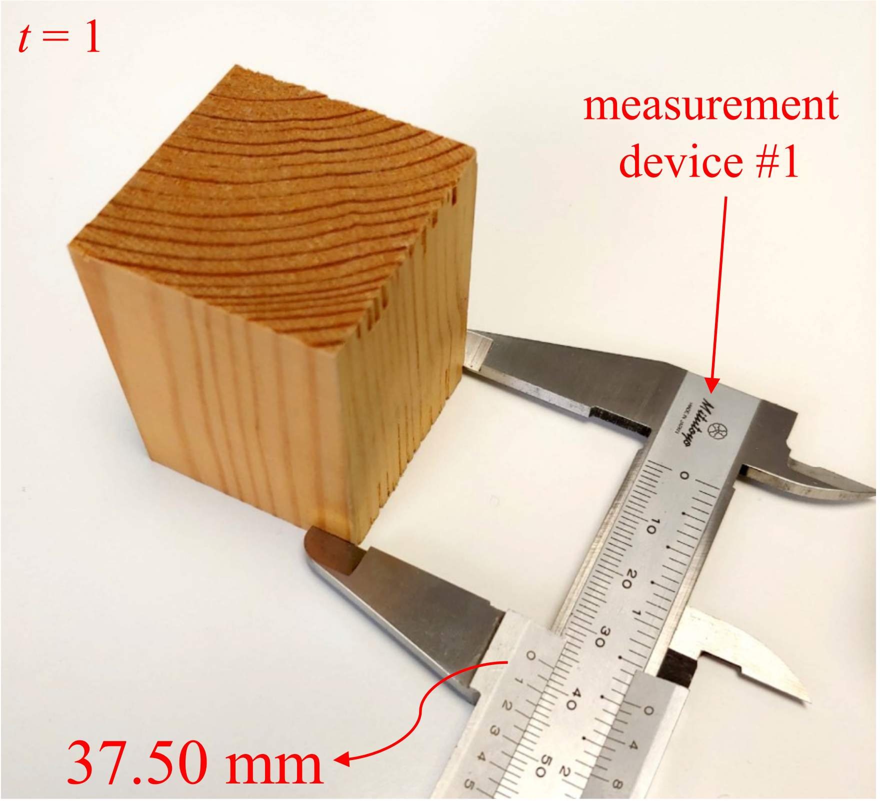
An interaction between a physical system ("measured system") and another physical system ("measurement device") that:

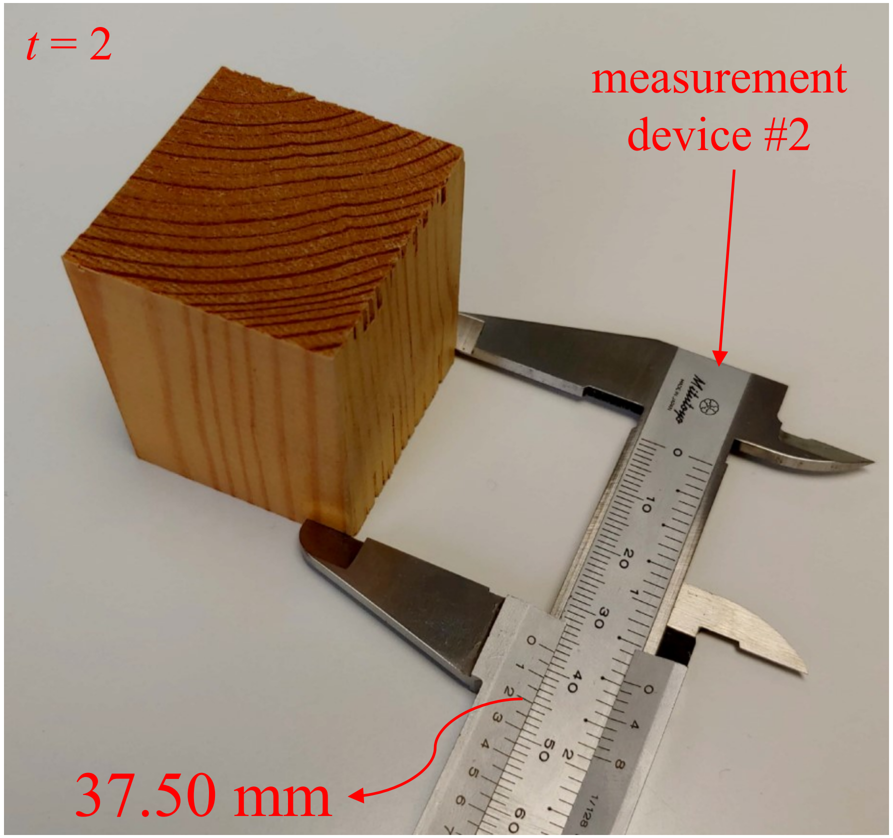
1. Yields the same outcome when performed consecutive times (even using a different copy of the measurement device)
2. Does not disturb any compatible observable
3. All its coarse-grainings can be implemented satisfying 1 and 2

Physical system + measurement devices

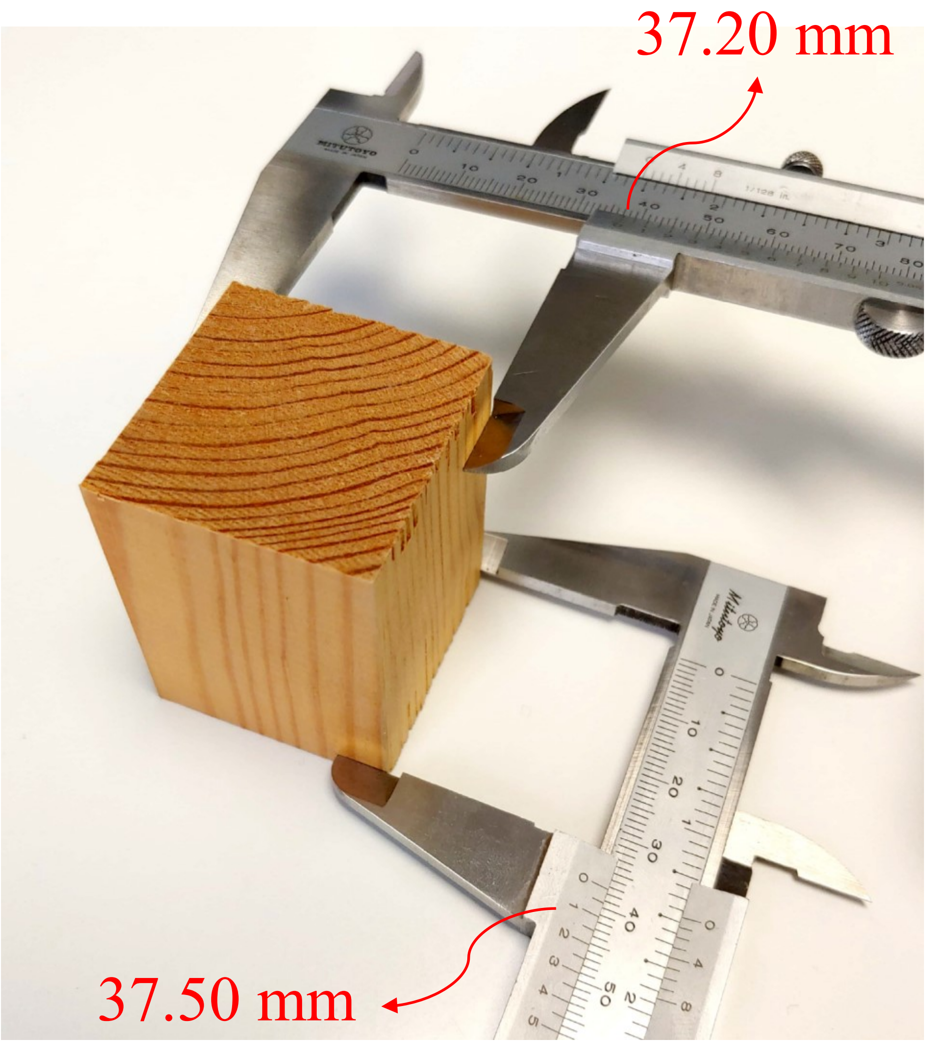


Yields the same outcome when repeated (w a diff. dev.)

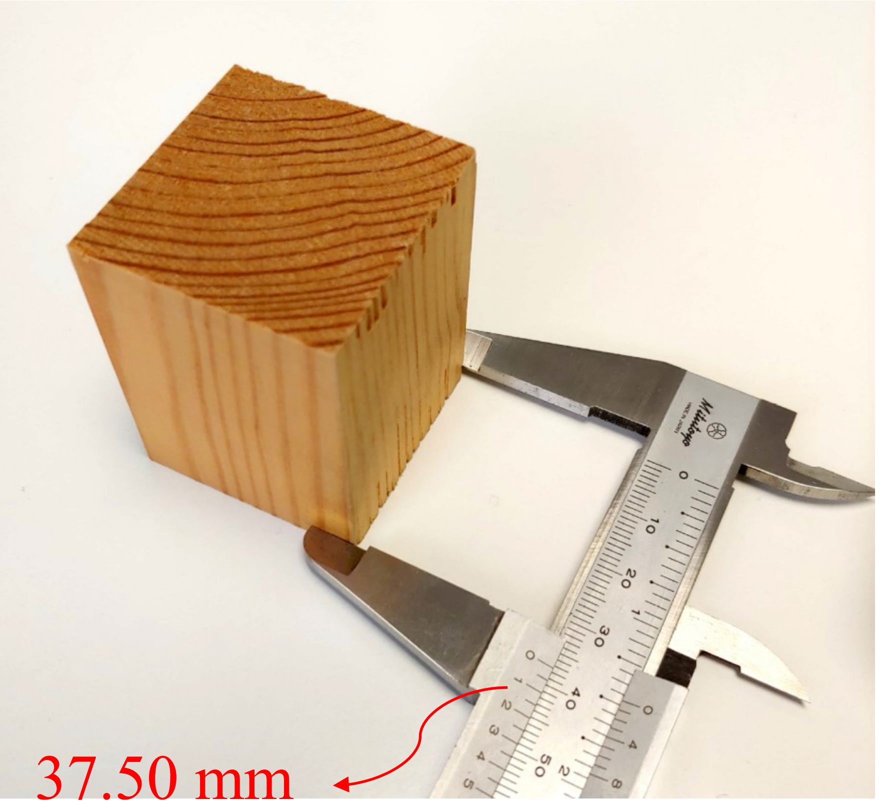




Does not disturb compatible observables



Performing or not the measurement on the top does not alter the outcome statistics of the measurment on the bottom



Compatible measurements

Two measurements x (with outcomes $a \in A$) and y (with outcomes $b \in B$) are compatible (or jointly measurable) if there is a measurement z (with outcomes (a, b), with $a \in A$ and $b \in B$), such that, for any state ψ and any outcome $a \in A$,

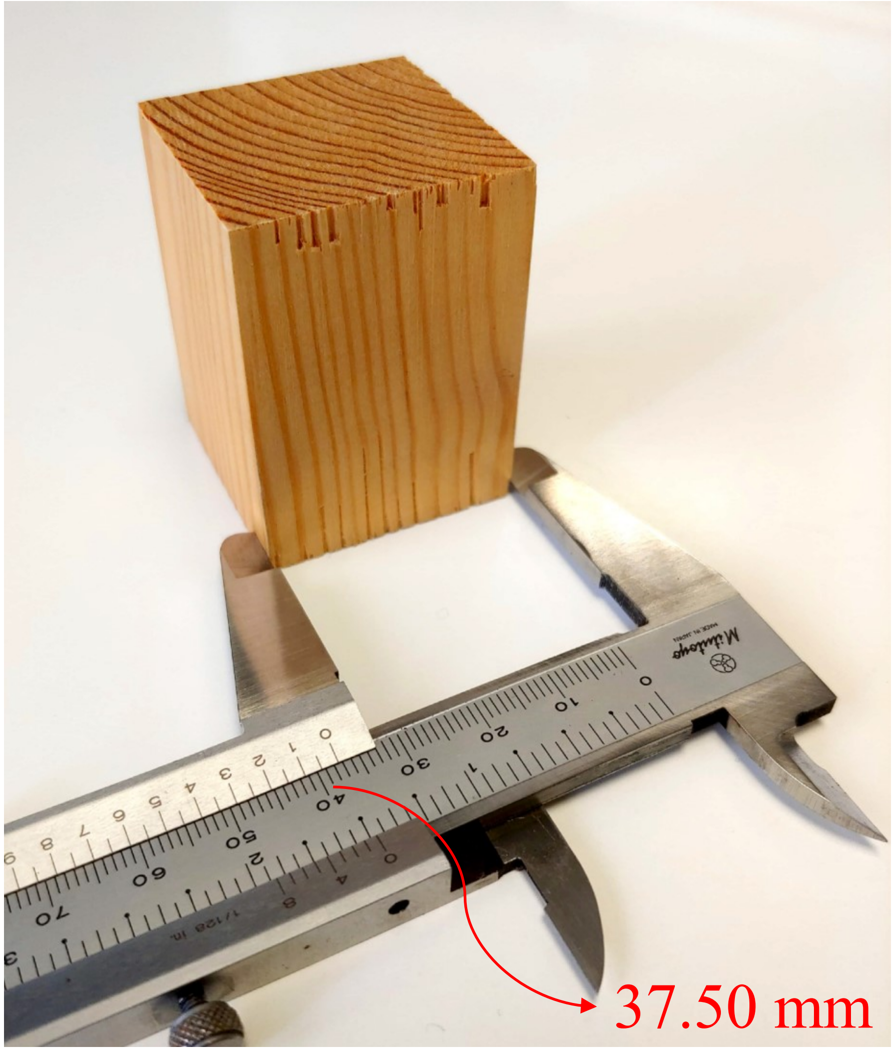
$$\sum_{b \in B} P(z = (a, b)|\psi) = P(x = a|\psi),$$

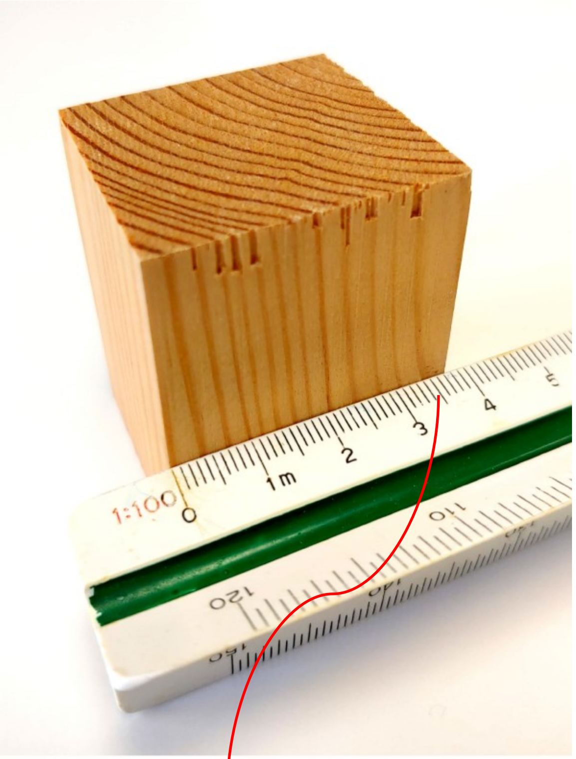
and, for any outcome $b \in B$,

$$\sum_{a \in A} P(z = (a, b)|\psi) = P(y = b|\psi),$$

where $P(x = a|\psi)$ is the probability of the event in which, when the system was in state ψ, measurement x has been performed and outcome a has been obtained.

Coarse-grainings have ideal implementations





37 mm

there is one outome for every 20 outcomes
of the measurement in the
left hand side

Coarse-graining

z with outcomes $c \in C$ is a coarse-graining of a measurement x with outcomes $a \in A$ if, for all $c \in C$, there is $A_c \subseteq A$ such that, for all states ψ,

$$P(z = c|\psi) = \sum_{a \in A_c} P(x = a|\psi)$$

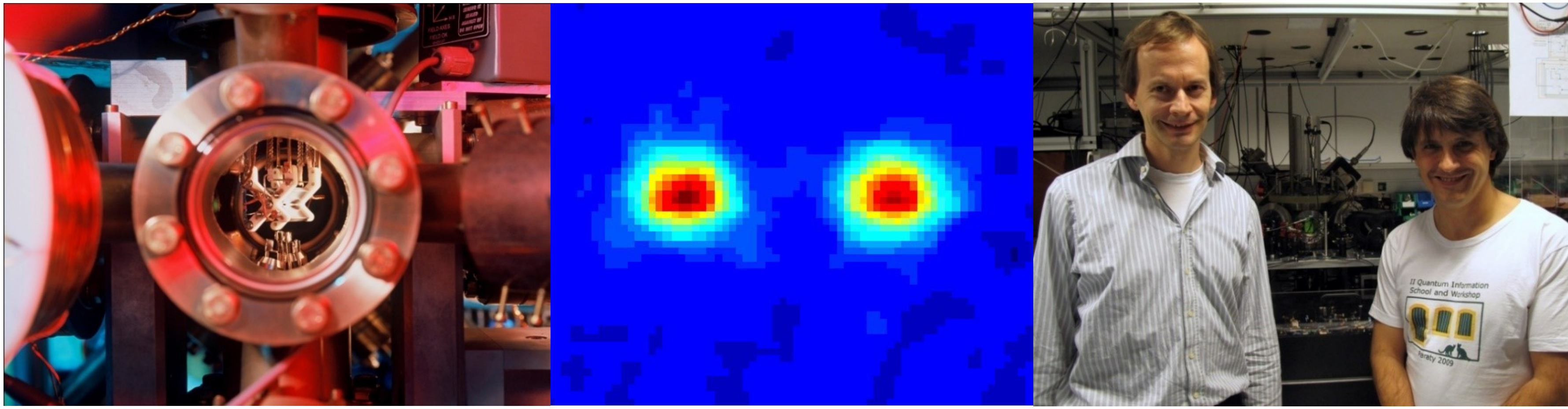
and $A_c \cap A_{c'} = \emptyset$ if $c \neq c'$.

Contextuality for ideal measurements

- It applies to ideal measurements
- It applies to outcome non-contextual models (rather than to models satisfying quantum constraints)
- It does not require assuming that coarse-grainings of two different measurements represent the same observable. For any observable, the same experimental device can be used in all contexts (see experiments with sequential measurements)

Contextuality experiments with sequential measurements

2009: Innsbruck contextuality experiment with ions



G. Kirchmair F. Zähringer R. Gerritsma M. Kleinmann O. Gühne R. Blatt C. Roos

G. Kirchmair, F. Zähringer, R. Gerritsma, M. Kleinmann, O. Gühne, A. Cabello, R. Blatt, and C. F. Roos, Nature **460**, 494 (2009).

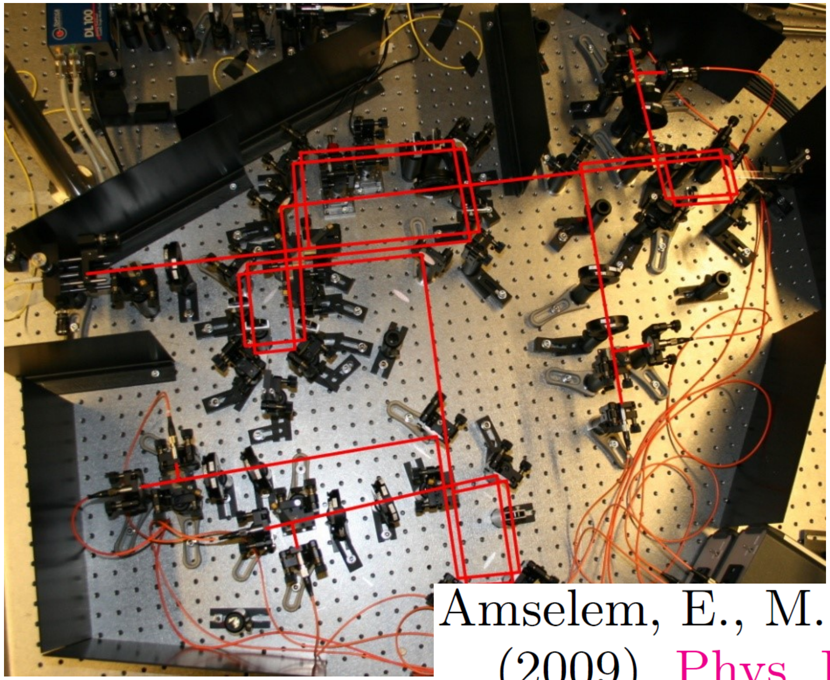
2009: Stockholm experiment with single photons

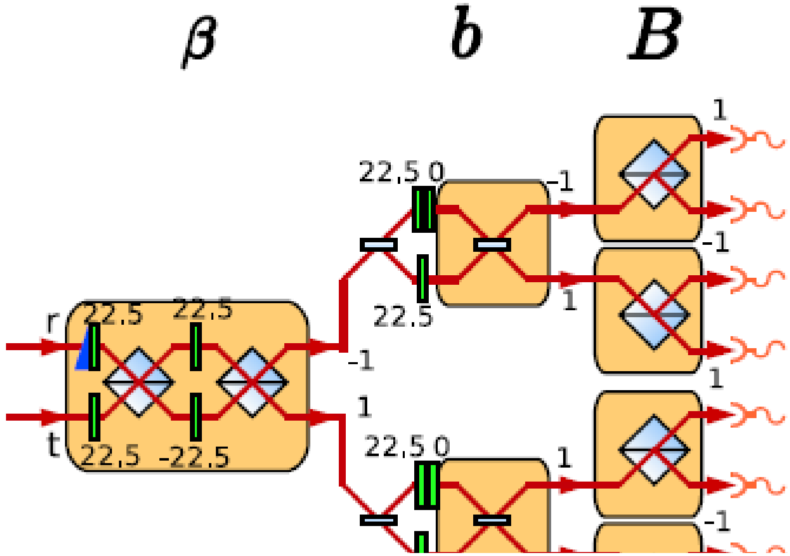
$$\langle ABC \rangle + \langle abc \rangle + \langle \alpha\beta\gamma \rangle + \langle Aa\alpha \rangle + \langle Bb\beta \rangle - \langle Cc\gamma \rangle \leq 4$$

$$A = \sigma_z^{(1)}, \quad B = \sigma_z^{(2)}, \quad C = \sigma_z^{(1)} \otimes \sigma_z^{(2)},$$

$$a = \sigma_x^{(2)}, \quad b = \sigma_x^{(1)}, \quad c = \sigma_x^{(1)} \otimes \sigma_x^{(2)},$$

$$\alpha = \sigma_z^{(1)} \otimes \sigma_x^{(2)}, \quad \beta = \sigma_x^{(1)} \otimes \sigma_z^{(2)}, \quad \gamma = \sigma_y^{(1)} \otimes \sigma_y^{(2)}.$$



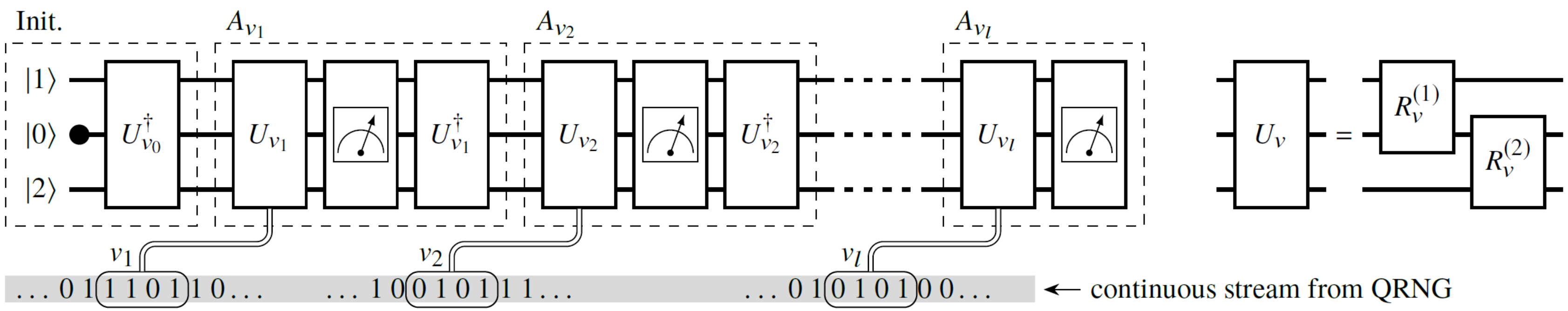


Amselem, E., M. Rådmark, M. Bourennane, and A. Cabello (2009), Phys. Rev. Lett. **103**, 160405.

2018: Sustained state-independent contextuality

- “Repeatable tests exist mostly in the imagination of theorists”

  A. Peres, *Quantum Theory: Concepts and Methods* (Kluwer, 1993), p. 29.

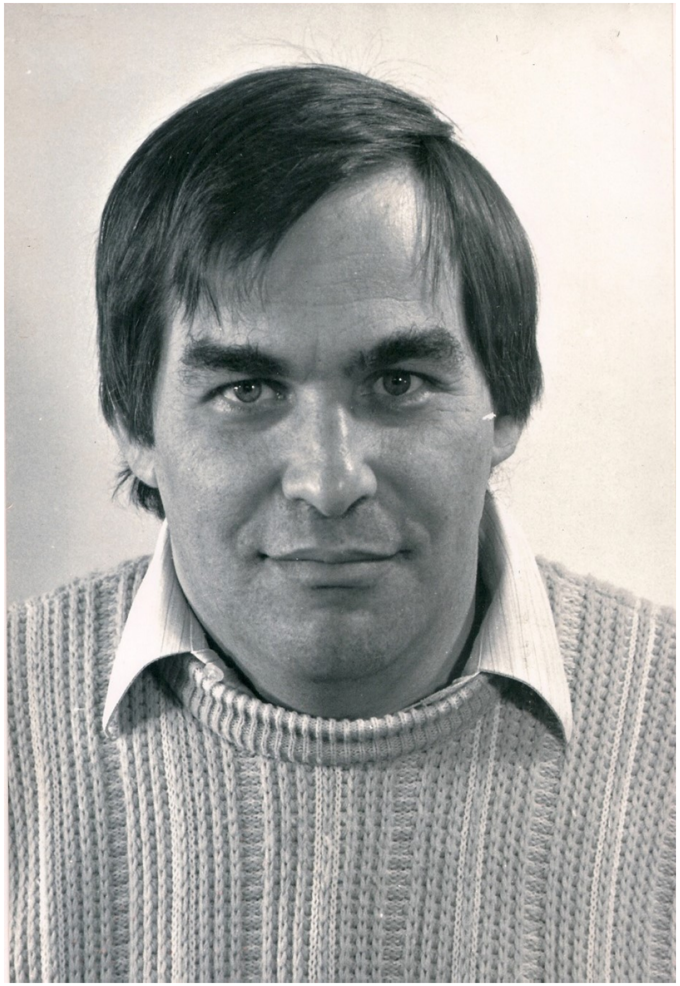


- 53 million (randomly chosen) sequential measurements on a single ion. Repeatability > 99.6%

F. M. Leupold, M. Malinowski, C. Zhang, V. Negnevitsky, A. Cabello, J. Alonso and J. P. Home, Phys. Rev. Lett. **120**, 180401 (2018).

The hidden history of non-contextuality inequalities

1994: Bell inequalities are Boole's conditions



Brit. J. Phil. Sci. **45** (1994), 95–125 *Printed in Great Britain*

George Boole's 'Conditions of Possible Experience' and the Quantum Puzzle

ITAMAR PITOWSKY*

ABSTRACT

In the mid-nineteenth century George Boole formulated his 'conditions of possible experience'. These are equations and inequalities that the relative frequencies of (logically connected) events must satisfy. Some of Boole's conditions have been rediscovered in more recent years by physicists, including Bell inequalities, Clauser Horne inequalities, and many others. In this paper, the nature of Boole's conditions and their relation to propositional logic is explained, and the puzzle associated with their violation by quantum frequencies is investigated in relation to a variety of approaches to the interpretation of quantum mechanics.

* While preparing this paper for publication I have learnt of the untimely death of Professor J. S. Bell, and I wish to dedicate the paper to his memory. This research was undertaken while I spent a sabbatical leave at Wolfson College, and the History and Philosophy of Science Department at the University of Cambridge. I would like to thank Michael Redhead and Jeremy Butterfield for their hospitality and for helpful discussions. A first draft of this paper has been distributed among the participants of the conference 'Einstein in Context' which was held in Israel, in April 1990. I have benefited from the comments of many colleagues. I would like to thank in particular Arthur Fine who enlightened me on the prism models, David Albert, Maya Bar-Hillel, Yemima Ben-Menachem, Mara Beller, Simon Saunders, and Mark Steiner. This research is partially supported by the Edelstein Center for the History and Philosophy of Science at the Hebrew University.

I. Pitowsky, Correlation polytopes: Their geometry and complexity, *Mathematical Programming* A50 (1991), 395–414.

Which of Boole's inequalities can be violated?

1963: Vorob'yev's theorem

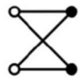
- A violation of Boole's inequalities can only occur for contextuality scenarios in which the relations of compatibility can be encoded in a graph (in which vertices represent measurements and edges represent relations of mutual compatibility) which is not chordal (i.e., it does not contain induced cycles of size larger than three). Otherwise, there is always a joint probability distribution and, therefore, a non-contextual model

N. N. Vorob'yev, Markov measures and Markov extensions, Theory Probab. Appl. **8**, 420 (1963).

N. N. Vorob'yev, Coalition games, Theory Probab. Appl. **12**, 251 (1967).

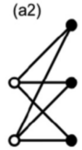
1963: Vorob'yev's theorem

(a1)

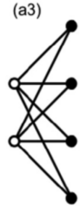


Bohm 1951's version of EPR
CHSH

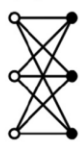
(a2)



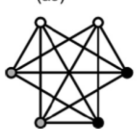
(a3)



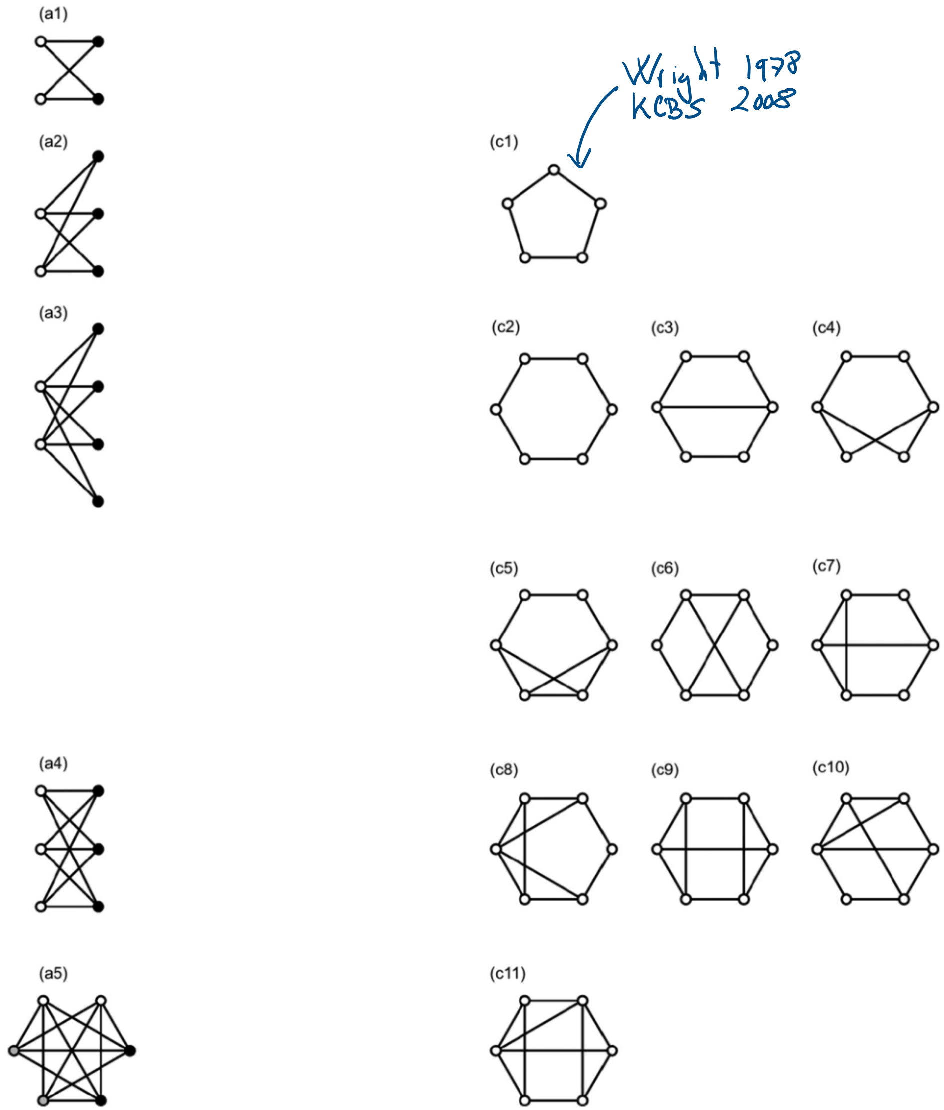
(a4)



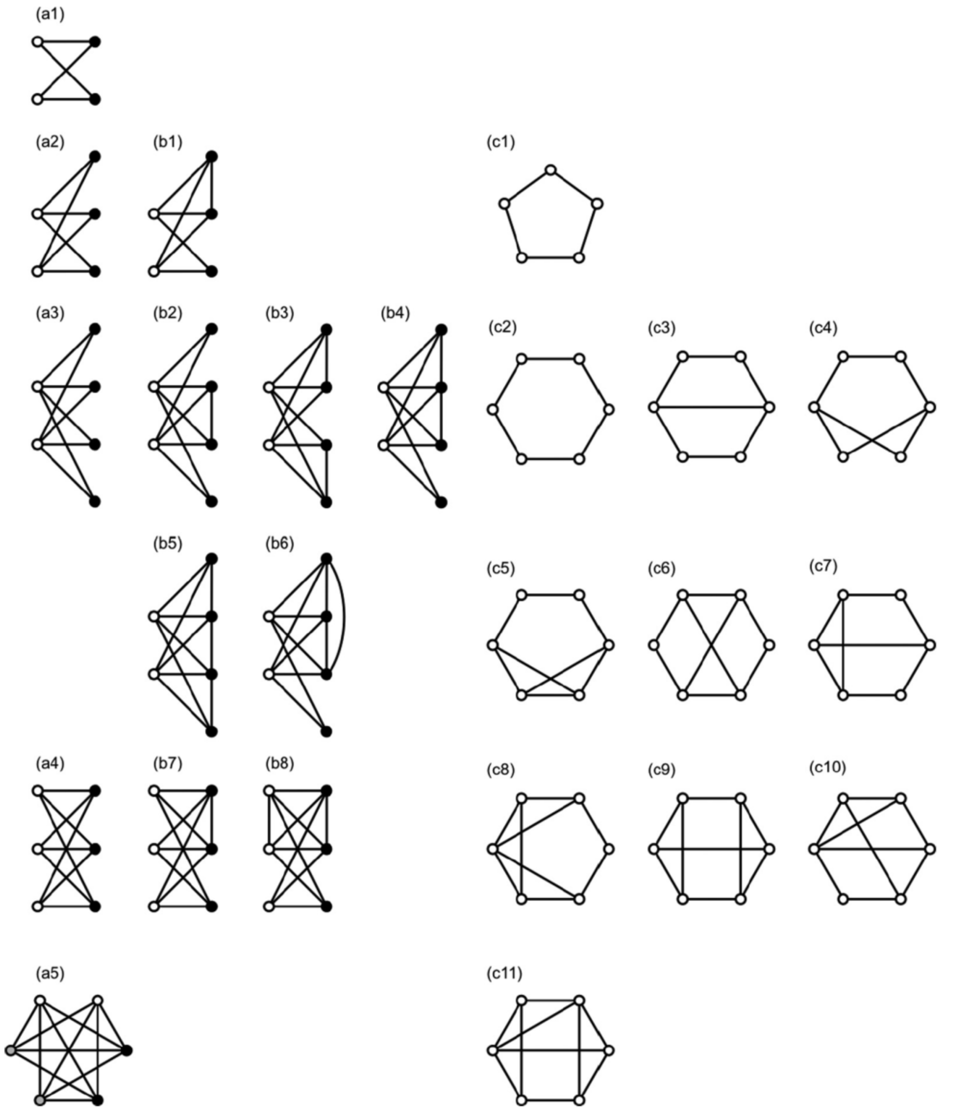
(a5)



1963: Vorob'yev's theorem



1963: Vorob'yev's theorem



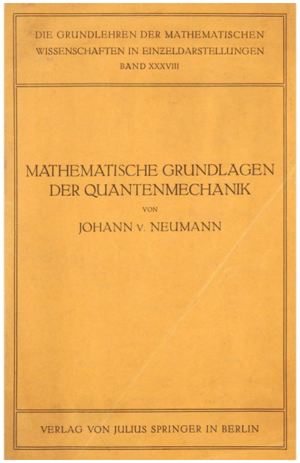
The perspective of GPTs

- The framework of Generalized Probabilistic Theories (GPTs) views QT as one possibility in a landscape of theories and asks whether nature could be “more crazy” than quantum

QT as a theory of ideal measurements

Incident #1

- Von Neumann's 1932 book is wrong about the post-measurement state
- The "right" state transformation corresponds to the only process that can be associated to a measurement of an observable *A* that does not disturb a subsequent measurement of any refined observable *B* (i.e., one where each outcome *b* is at most as likely as a certain outcome a_b of *A*, for all states)



G. Lüders, Über die Zustandsänderung durch den Meßprozeß, Ann. Phys. (Berlin) **443**, 322 (1950); [English translation Concerning the state-change due to the measurement process, Ann. Phys. (Berlin) **15**, 663 (2006)].

Incident #2

- POVMs are the most general type of quantum measurements
- However, every POVM can be realized as a PVM in a Hilbert space of augmented dimension
- “Generalized” quantum measurements do not produce correlations that cannot be produced by ideal measurements

M. A. Neumark, Izv. Akad. Nauk S.S.S.R. [Bull. Acad. Sci. U.S.S.R.] Sér. Mat. **4**, 53 (1940); Izv. Akad. Nauk S.S.S.R. [Bull. Acad. Sci. U.S.S.R.] Sér. Mat. **4** 277 (1940); C.R. (Dokl.) Acad. Sci. U.R.S.S. (N.S.) **41** 359 (1943).

The perspective of GPTs

- The framework of Generalized Probabilistic Theories (GPTs) views QT as one possibility in a landscape of theories and asks whether nature could be “more crazy” than quantum
- Some of these theories differ from QT in observable aspects
- E.g., in the set of correlations for Bell scenarios
- We have not identified a principle that explains the quantum set of correlations for Bell scenarios. Nature could be “more Bell nonlocal” than it is
- QT is a probability theory for ideal measurements
- Could nature be “more contextual” than it is for ideal measurements?

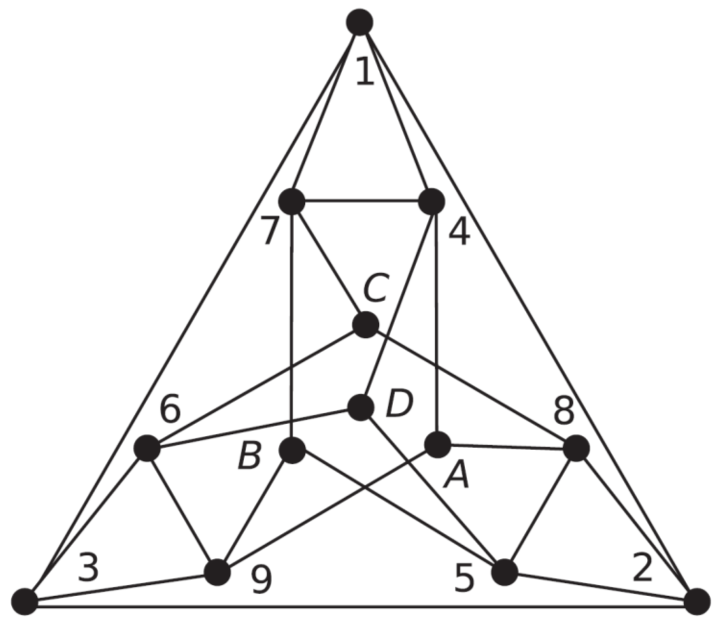
Other developments

2005: Spekkens' non-contextual ontological models

- Idea: Non-contextual ontological models must reproduce the statistical equivalence class structure of preparations, transformations, and unsharp measurements (rather than just of sharp/ideal measurements as in "KS contextuality")
- Question: Is this assumption as natural/well motivated as it is for sharp/ideal measurements?
- Observation: Experimental imperfections might imply that, in practice, no two experimental procedures are found to be operationally equivalent, in which case the assumption of non-contextuality is never applicable. (Similarly, in KS contextuality, experimental imperfections may make difficult to implement ideal measurements). Relaxations supplemented by quantitative measures of similarity in the space of procedures, and corresponding measures in the space of their ontological representations are needed for experimental tests

Spekkens, R. W. (2005), Phys. Rev. A **71**, 052108.

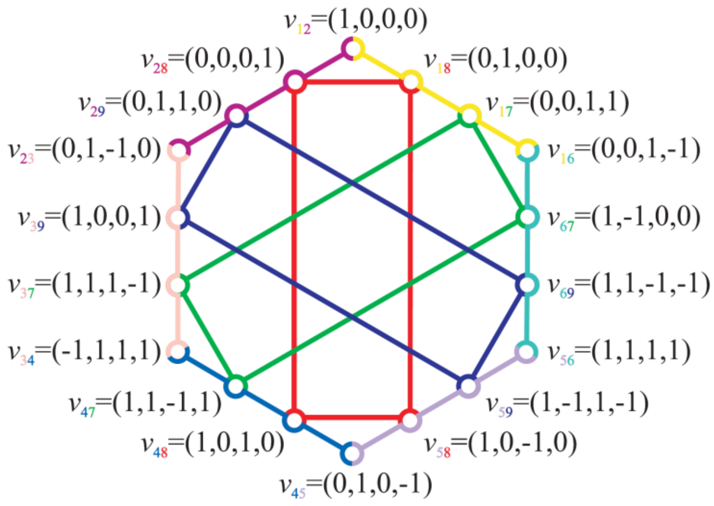
2012: Yu-Oh inequality

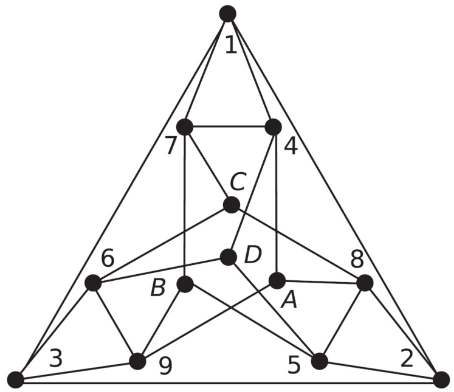


$$
\begin{array}{lll}
v_1 = (1,0,0) & v_5 = (1,0,-1) & v_A = (-1,1,1) \\
v_2 = (0,1,0) & v_6 = (1,-1,0) & v_B = (1,-1,1) \\
v_3 = (0,0,1) & v_7 = (0,1,1) & v_C = (1,1,-1) \\
v_4 = (0,1,-1) & v_8 = (1,0,1) & v_D = (1,1,1)
\end{array}
$$

Yu, S., and C. H. Oh (2012), Phys. Rev. Lett. **108** (3), 030402.

2016-2020: Minimal KS and SIC sets





$v_1 = (1,0,0)$ $v_5 = (1,0,-1)$ $v_A = (-1,1,1)$
$v_2 = (0,1,0)$ $v_6 = (1,-1,0)$ $v_B = (1,-1,1)$
$v_3 = (0,0,1)$ $v_7 = (0,1,1)$ $v_C = (1,1,-1)$
$v_4 = (0,1,-1)$ $v_8 = (1,0,1)$ $v_D = (1,1,1)$

Cabello, A., J. M. Estebaranz, and G. García-Alcaine (1996), Phys. Lett. A **212** (4), 183.

Yu, S., and C. H. Oh (2012), Phys. Rev. Lett. **108** (3), 030402.

Cabello, A., M. Kleinmann, and J. R. Portillo (2016a), J. Phys. A: Math. Theor. **49**, 38LT01.

Xu, Z.-P., J.-L. Chen, and O. Gühne (2020), Phys. Rev. Lett. **124**, 230401.

Contextuality as a resource

2009: Computational power of contextual correlations

Anders, J., and D. E. Browne (2009a), Phys. Rev. Lett. **102**, 050502.

2013: Contextuality in measurement-based q. comp.

Raussendorf, R. (2013b), Phys. Rev. A **88**, 022322.

2014: Contextuality in q. comp. via magic sates

Howard, M., J. Wallman, V. Veitch, and J. Emerson (2014), Nature (London) **510**, 351.

Contextuality in quantum computation. Recommended

- D. Browne, “Contextuality and non-contextuality in (qudit) quantum computing” (video): http://pirsa.org/displayFlash.php?id=17070053
- M. Howard, “Magic sates and contextuality” (slides): https://www.cs.ox.ac.uk/conferences/contextuality/slidesMarkHoward.pdf

2005: Contextuality + no-signalling = secure key

Ekert, A. K. (1991), Phys. Rev. Lett. **67**, 661.

Barrett, J., L. Hardy, and A. Kent (2005), Phys. Rev. Lett. **95**, 010503.

2006: Contextuality + no-sig. = private randomness

Colbeck, R. (2006), *Quantum and Relativistic Protocols for Secure Multi-Party Computation*, Ph.D. thesis (University of Cambridge), arXiv:0911.3814.

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- The framework of Generalized Probabilistic Theories (GPTs) views QT as one possibility in a landscape of theories and asks whether nature could be "more crazy" than quantum
- Some of these theories differ from QT in observable aspects
- E.g., in the set of correlations for ideal measurements
- However, nature could not be "more KS contextual"

2019: QT is the most contextual GPT for ideal measurements

- Which Boole's inequalities can be violated?
  - Answer: See Vorob'yev's theorem. Quantum theory violates all the inequalities that can be violated
- What is the largest set of correlations for a KS scenario?
  - Answer: The quantum one (assuming that statistically independent copies of any behavior exist and that the theory yields behaviors for any scenario)
- How this compares with quantum theory?
  - Answer: Nature could not be "more KS contextual", quantum theory is the most contextual GPT for ideal measurements

Cabello, A. (2019b), Phys. Rev. A **100**, 032120.

What does it mean?

- Quantum contextuality is a signature of an *ontological* absence of constraints in the way certain parts of the world interact

Cabello, A. (2019a), Philos. Trans. R. Soc. A **377**, 2019.0136.

- Quantum contextuality simply follows from adopting a Bayesian framework to organize beliefs and update them when new information becomes available

Chiribella, G., A. Cabello, M. Kleinmann, and M. P. Müller (2020), Phys. Rev. Research **2**, 042001.

Thank you!