The future of quantum contextuality

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Y COMPUTACIONAL

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NUS virtual seminar
November 17, 2020

The future of quantum contextuality

Aim of this talk

- Review some results on quantum contextuality

  1. The Kochen-Specker theorem
  2. Non-contextuality inequalities
  3. Nature cannot be “more contextual” for ideal measurements
  4. Nature allow for “absolute maximal contextuality”
  5. Contextuality is needed for quantum computational speedup
  6. Classical simulation of quantum contextuality requires memory
  7. Contextuality enables nonlocality

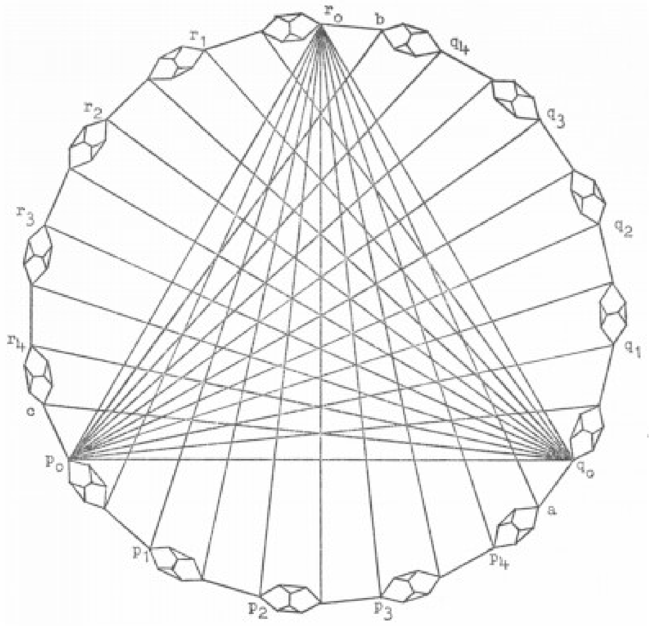
Aim of this talk

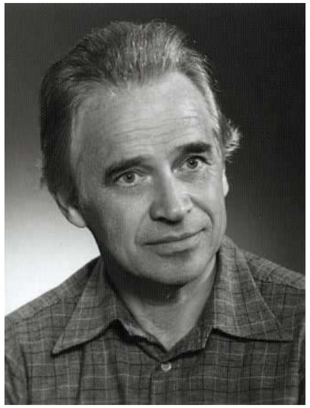
- List some open problems

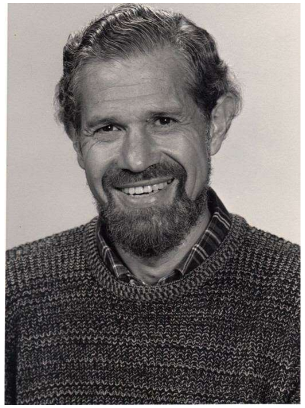
  1. Minimal KS set in dimension 3?
  2. Contextuality in unstudied scenarios
  3. Is contextuality the key property to understand nature?
  4. How to produce "absolute maximal contextuality"?
  5. Is contextuality needed in the circuit model?
  6. How to compute and maximize the memory cost(s)?
  7. How to convert contextuality into nonlocality?

Result #1

1967: The Kochen-Specker theorem







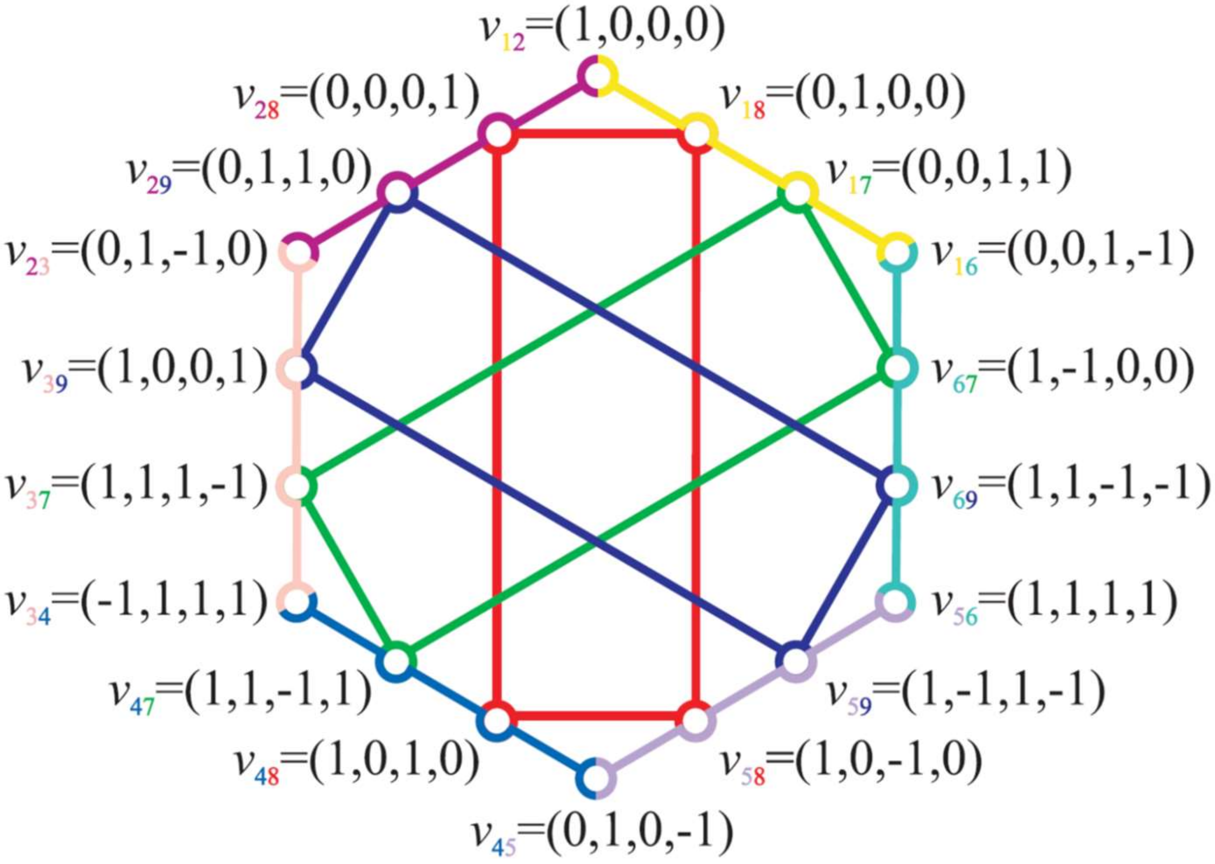
E. P. Specker, Adrian Specker, and S. Kochen at Rigiblick, Zürich, January 1963. Courtesy of Suzanne Specker

Kochen, S., and E. P. Specker (1965a), in *Proceedings of the 1964 International Congress for Logic, Methodology and Philosophy of Science, Jerusalem*, edited by Y. Bar-Hillel (North-Holland, Amsterdam) pp. 45–57.

Kochen, S., and E. P. Specker (1965b), in *Symposium on the Theory of Models: Proceedings of the 1963 International Symposium at Berkeley*, edited by J. W. Addison, L. Henkin, and A. Tarski (North-Holland, Amsterdam) pp. 177–189.

Kochen, S., and E. P. Specker (1967), J. Math. Mech. **17** (1), 59.

The 18-vector proof of the KS theorem



A. Cabello, J.M. Estebaranz, and G. García-Alcaine, Phys. Lett. A **212**, 183 (1996).

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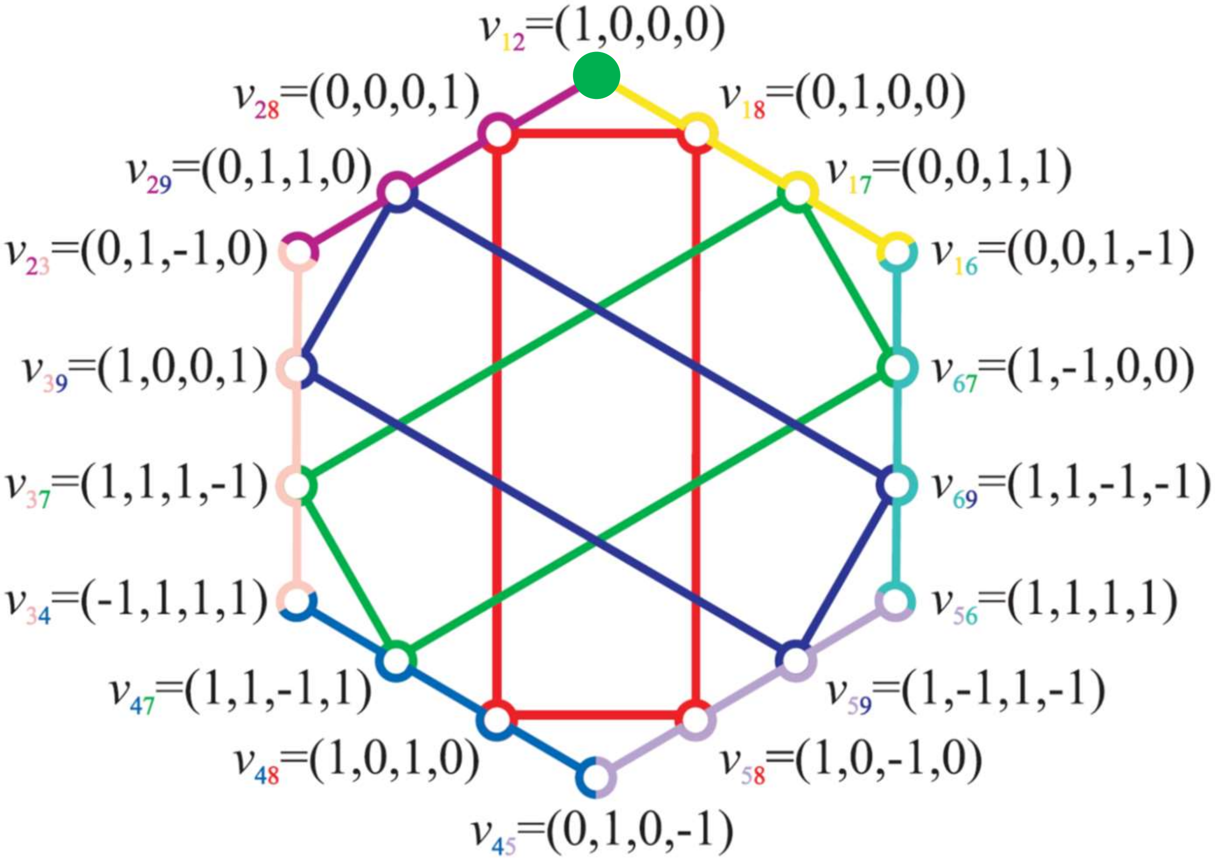

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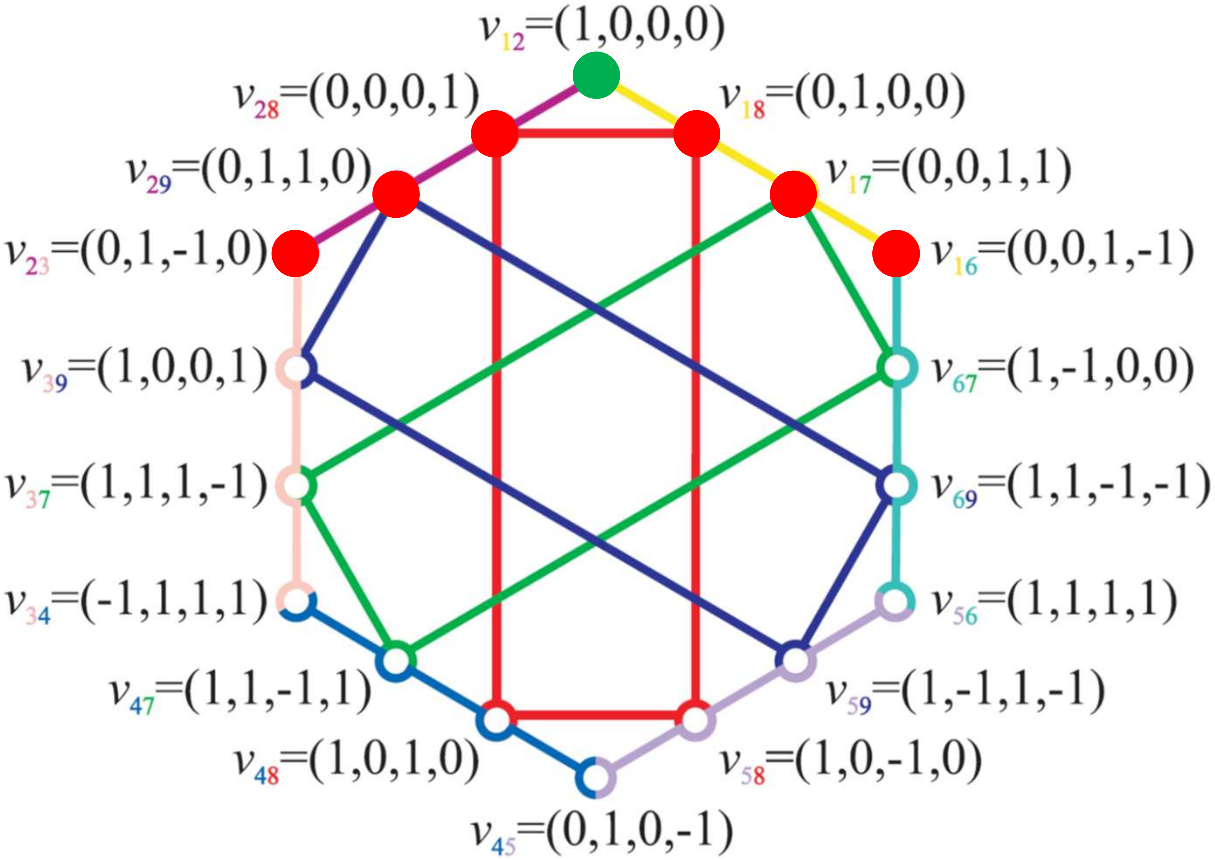
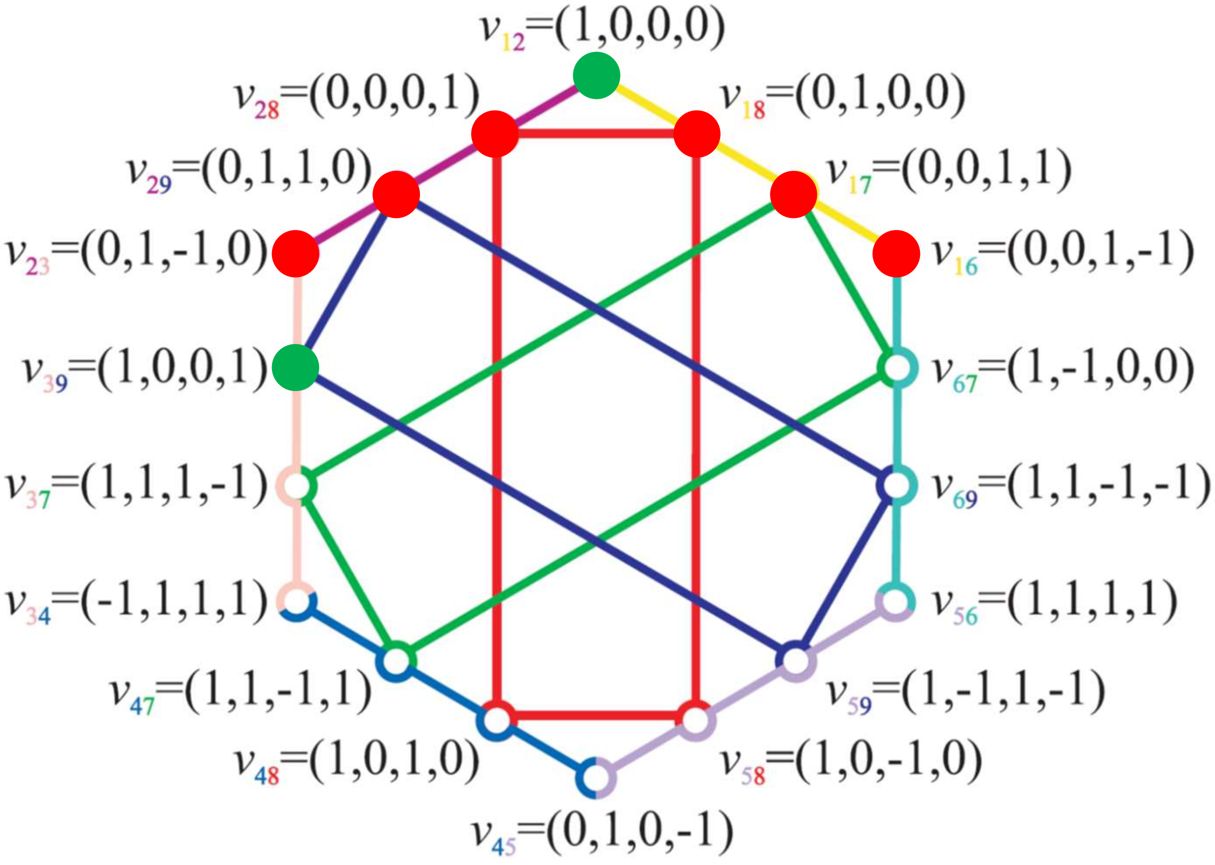


A. Cabello, J.M. Estebaranz, and G. García-Alcaine, Phys. Lett. A **212**, 183 (1996).

The 18-vector proof of the KS theorem



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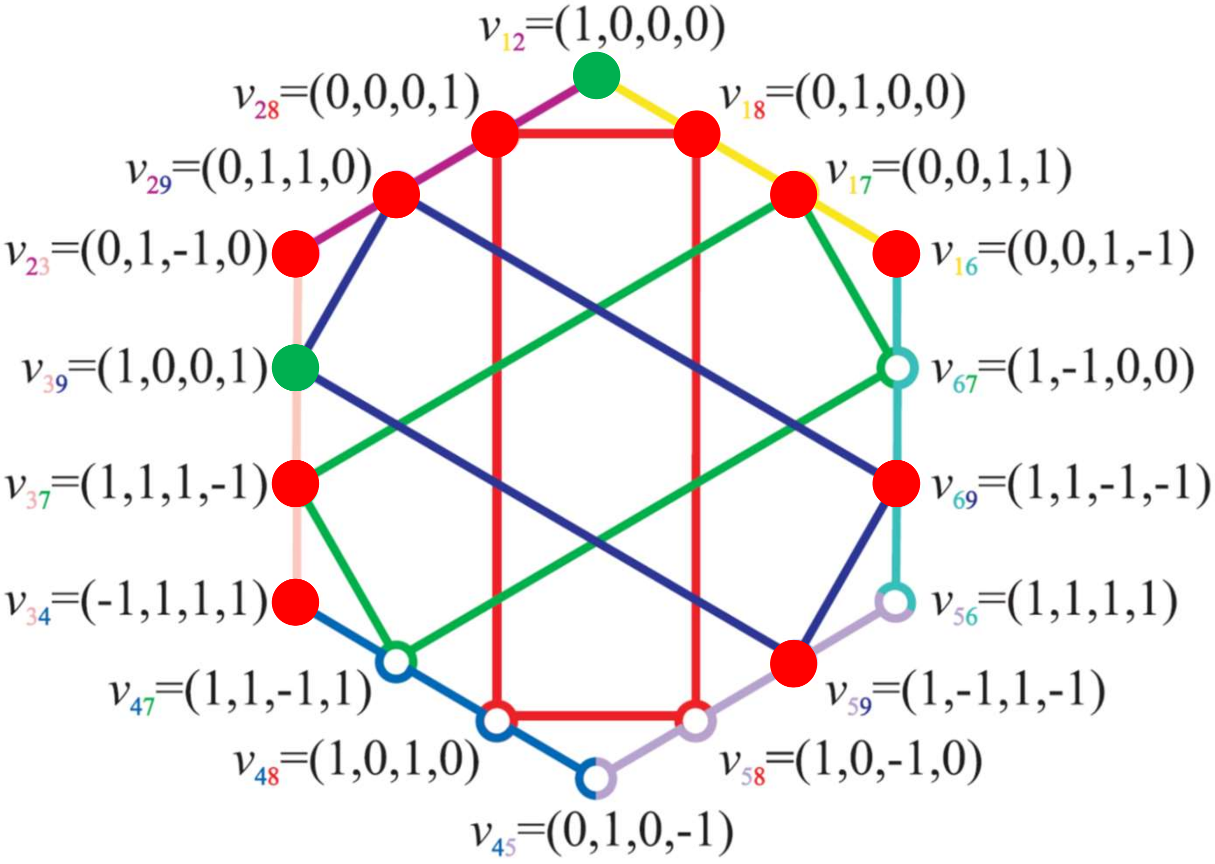

A. Cabello, J.M. Estebaranz, and G. García-Alcaine, Phys. Lett. A **212**, 183 (1996).

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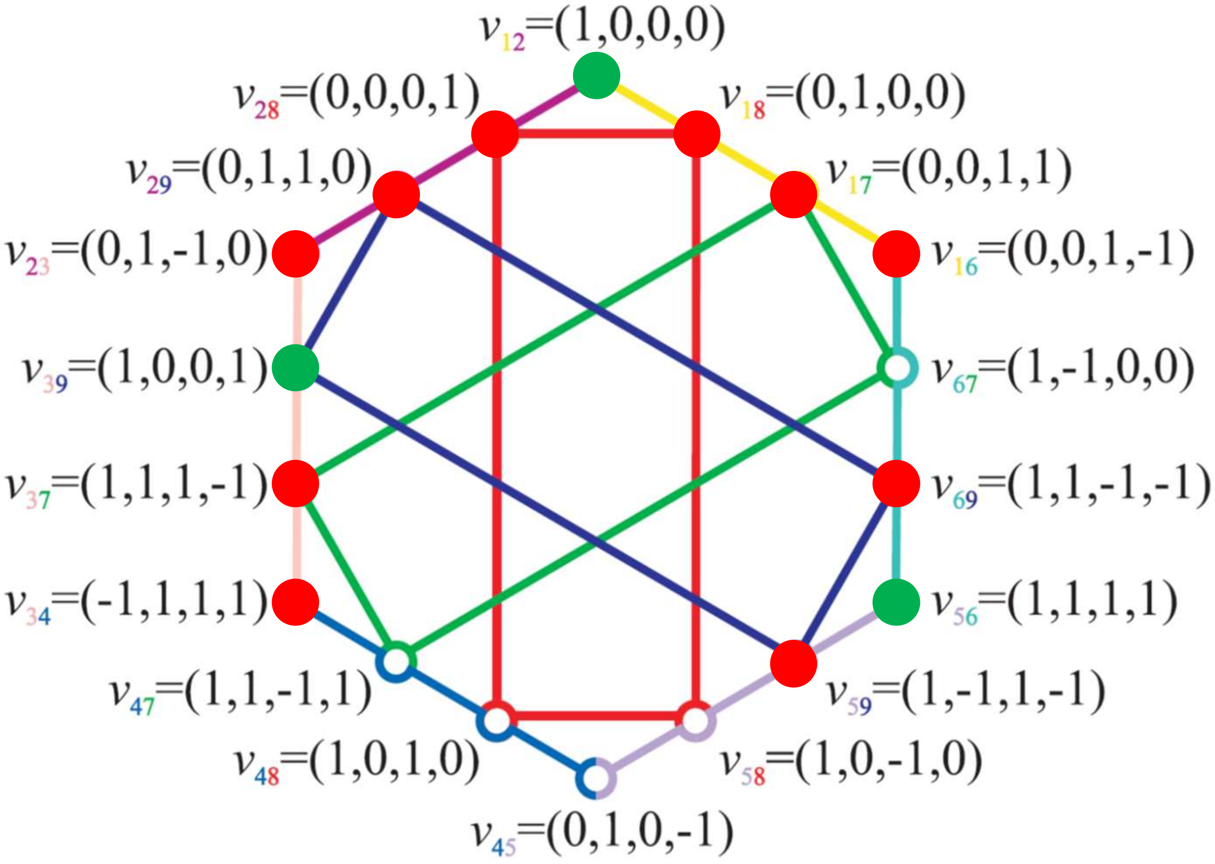

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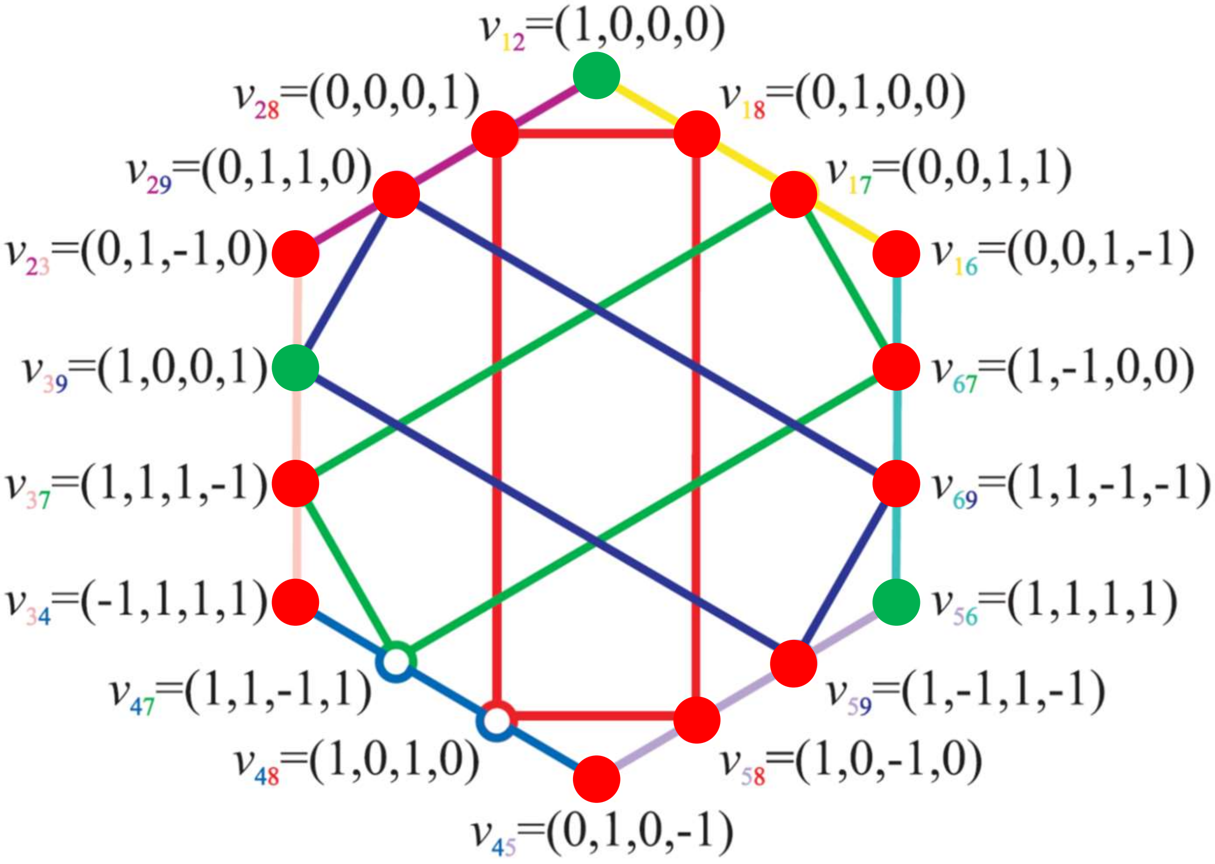

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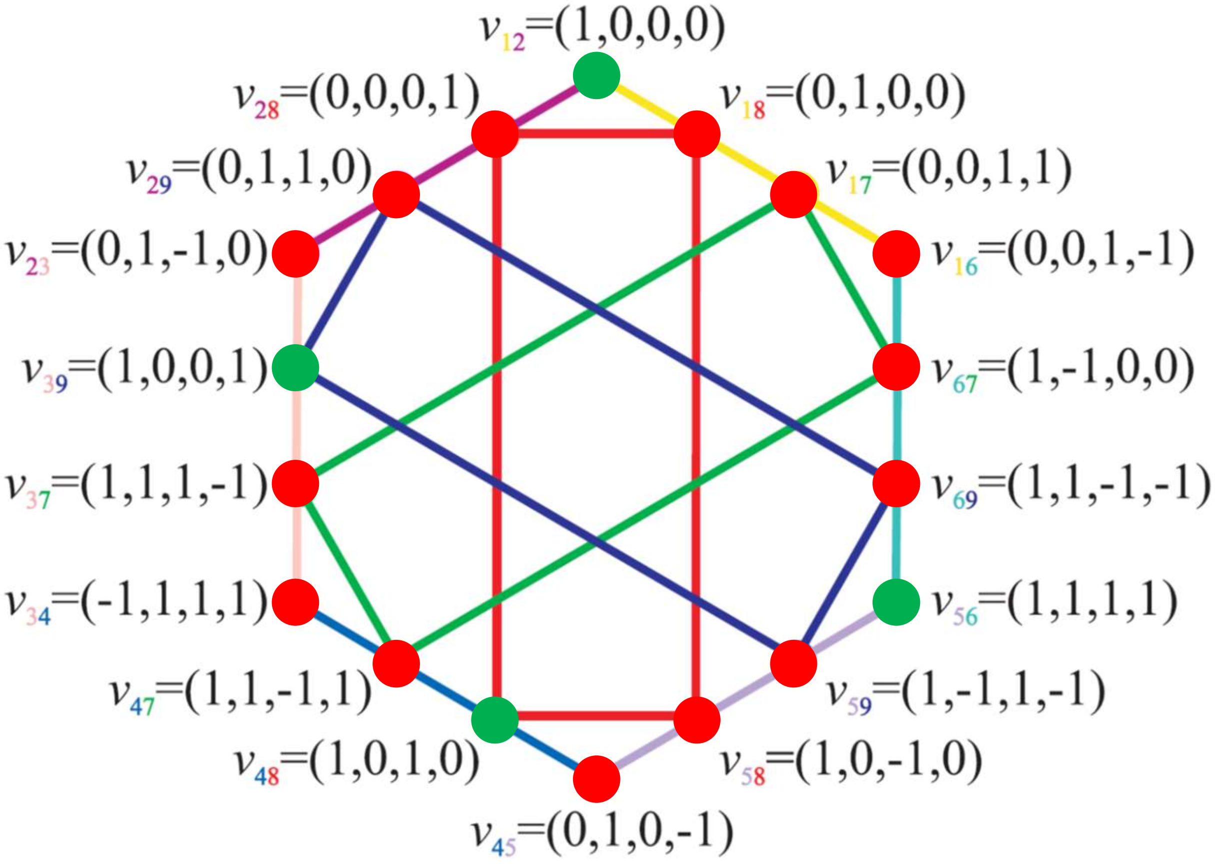

A. Cabello, J.M. Estebaranz, and G. García-Alcaine, Phys. Lett. A **212**, 183 (1996).

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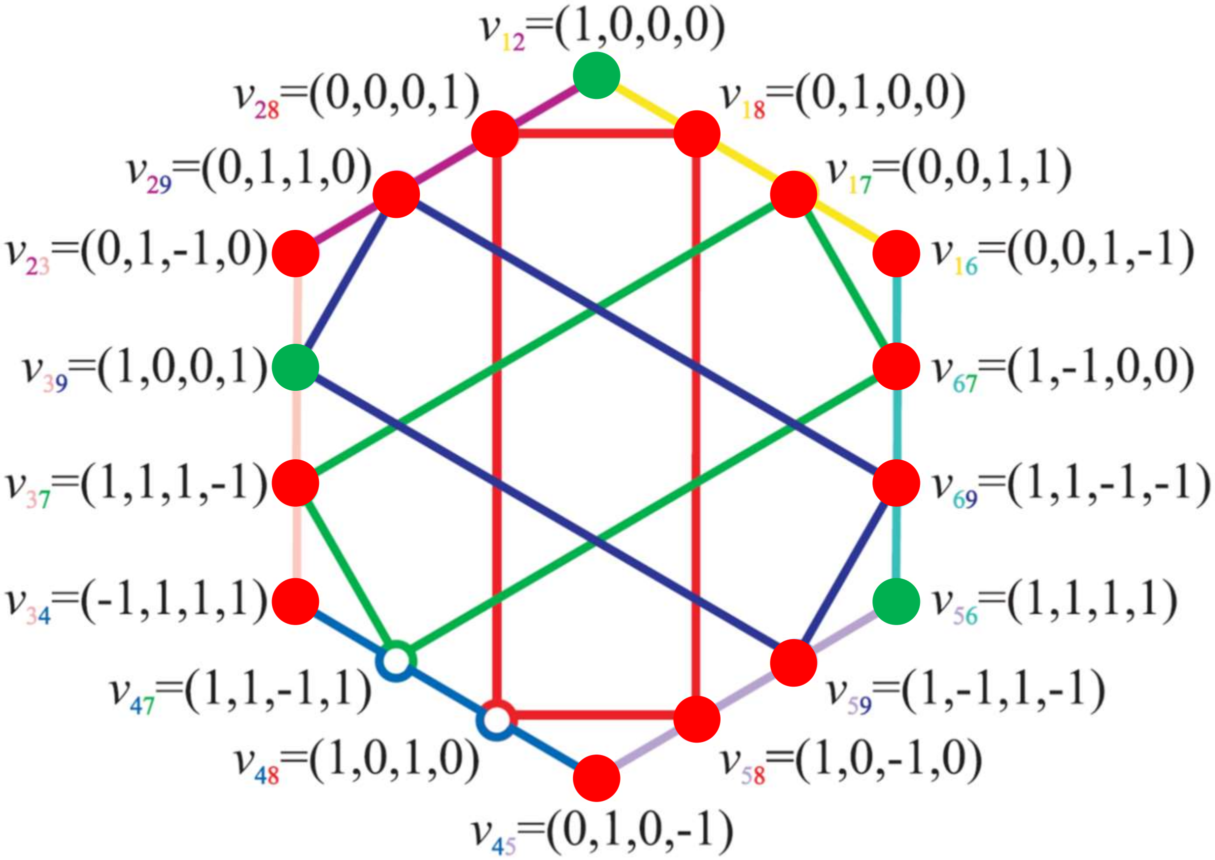

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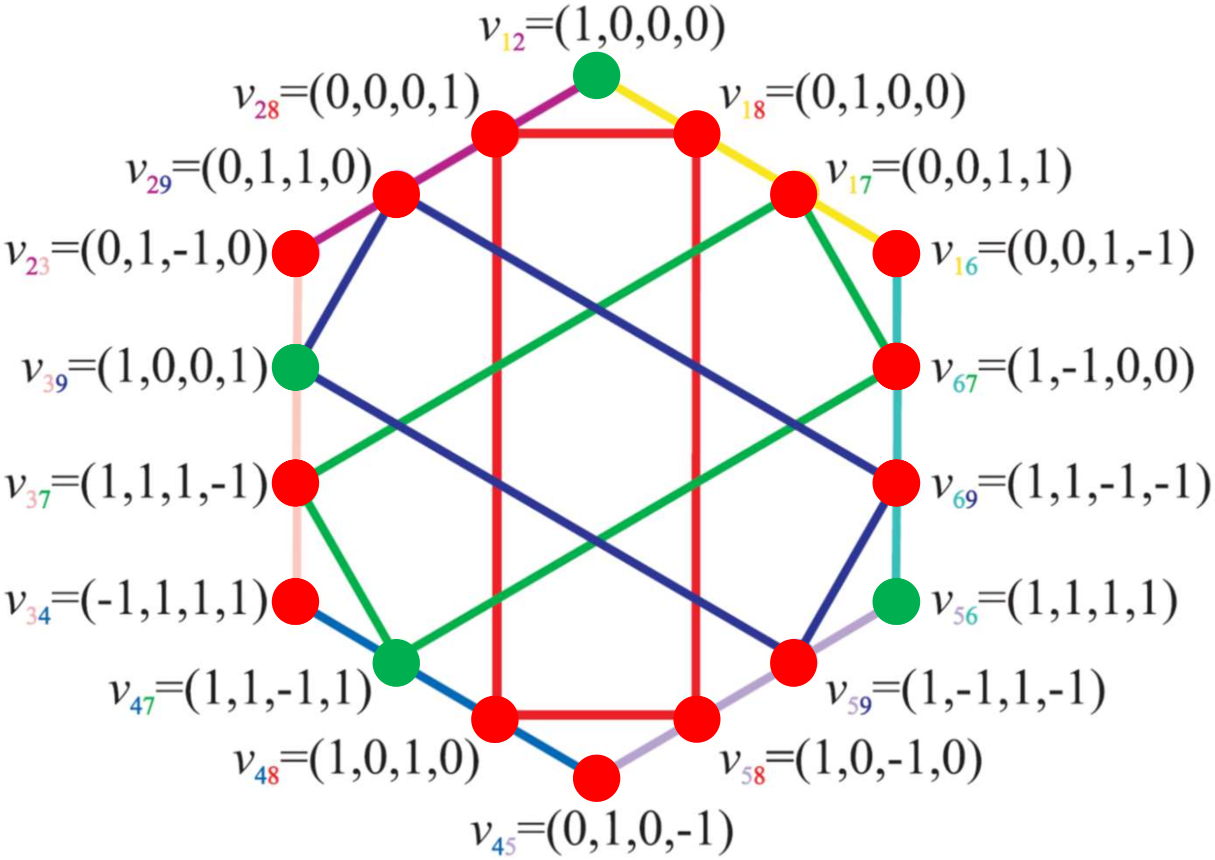
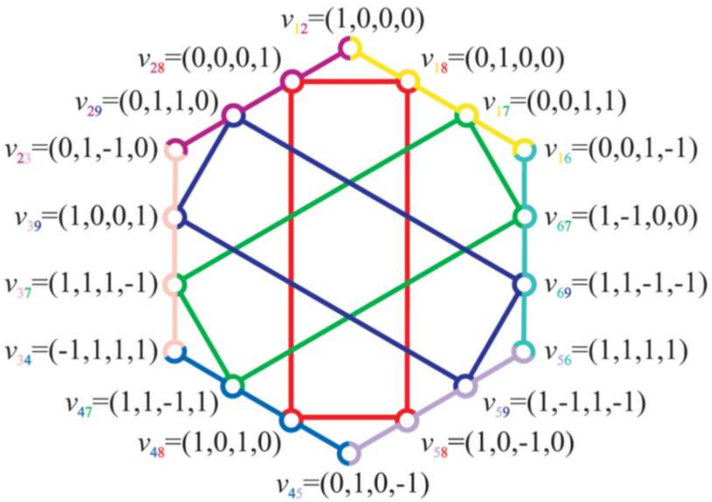
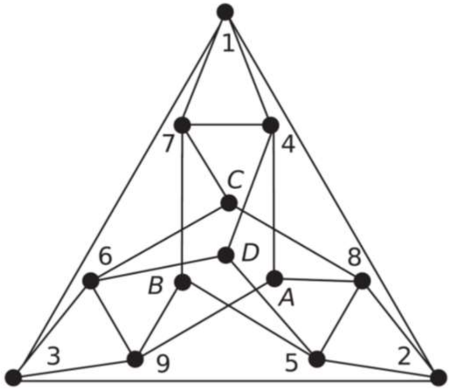


A. Cabello, J.M. Estebaranz, and G. García-Alcaine, Phys. Lett. A **212**, 183 (1996).

Problem #1

1996-2020: Minimal KS and SIC sets





$v_1 = (1,0,0)$ $v_5 = (1,0,-1)$ $v_A = (-1,1,1)$
$v_2 = (0,1,0)$ $v_6 = (1,-1,0)$ $v_B = (1,-1,1)$
$v_3 = (0,0,1)$ $v_7 = (0,1,1)$ $v_C = (1,1,-1)$
$v_4 = (0,1,-1)$ $v_8 = (1,0,1)$ $v_D = (1,1,1)$

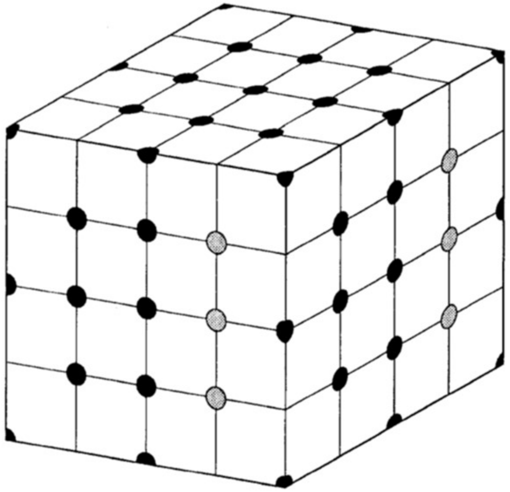
Cabello, A., J. M. Estebaranz, and G. García-Alcaine (1996), Phys. Lett. A **212** (4), 183.

Yu, S., and C. H. Oh (2012), Phys. Rev. Lett. **108** (3), 030402.

Cabello, A., M. Kleinmann, and J. R. Portillo (2016a), J. Phys. A: Math. Theor. **49**, 38LT01.

Xu, Z.-P., J.-L. Chen, and O. Gühne (2020), Phys. Rev. Lett. **124**, 230401.

Open problem: Minimal KS SET in dim 3?



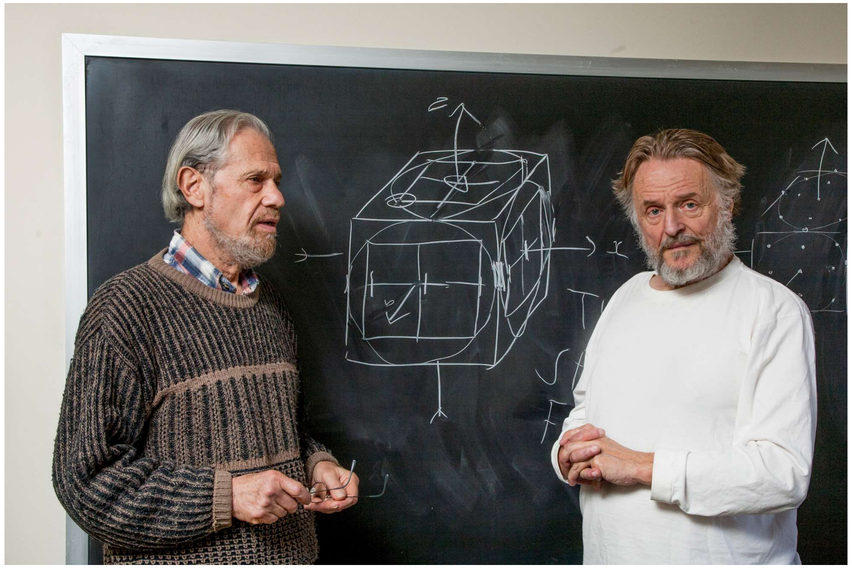


Plate II. The Kochen-Specker theorem, discussed in Chapter 7, is of fundamental importance for quantum theory. Its most "economical" proof makes use of 31 rays, which form 17 orthogonal triads (see Exercise 7.20, page 211). These rays are obtained by connecting the center of the cube to the black dots on its faces and edges (the six gray dots are not used in that proof).

Peres, A. (1993), *Quantum Theory: Concepts and Methods* (Kluwer, Dordrecht).

Result #2

Unifying Bell and KS theorems?

- Bell's theorem leads to experimental tests of whether the world can be explained with theories which can be defined without any reference to quantum mechanics. In them, any observable is measured with the same device in every context

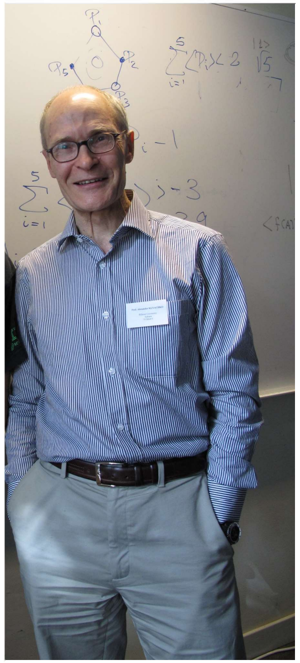
- The KS theorem is attached to quantum mechanics:

  - The KS theorem does not refer to general measurements, but to those that are represented, in quantum mechanics, by self-adjoint operators. (There are other measurements in quantum mechanics)

  - The proof of the KS theorem includes constraints that are specific to quantum systems. (E.g., KS, PM)

  - The experimental translation of the KS theorem (as proposed by KS and Bell) assumes quantum mechanics, as it is assumed that coarse-grainings of two different (and incompatible) measurements represent the same observable based on the fact that, in quantum mechanics, both yield the same outcome statistics

2008: The KCBS inequality



$$-\langle C_1C_2\rangle - \langle C_2C_3\rangle - \langle C_3C_4\rangle - \langle C_4C_5\rangle - \langle C_5C_1\rangle \overset{\text{NCHV}}{\leq} 3 \overset{\text{QT}}{\leq} 4\sqrt{5}-5 \approx 3.944.$$

A. A. Klyachko, in *Physics and Theoretical Computer Science: From Numbers and Languages to (Quantum) Cryptography*, edited by J.-P. Gazeau, J. Nešetřil, and B. Rovan, NATO Security Through Science Series: Information and Communication Security vol. 7 (IOP Press, London, 2007), p. 25.
A. A. Klyachko, M. A. Can, S. Binicioğlu, and A. S. Shumovsky, Phys. Rev. Lett. **101**, 020403 (2008).

2008: The Peres-Mermin inequality

$$\boxed{\langle ABC\rangle+\langle abc\rangle+\langle \alpha\beta\gamma\rangle+\langle Aa\alpha\rangle+\langle Bb\beta\rangle-\langle Cc\gamma\rangle \leq 4}$$

- The inequality holds under the assumption of outcome non-contextuality
- Quantum mechanics is not assumed in any way

Cabello, A. (2008), Phys. Rev. Lett. **101** (21), 210401.

2008: The Peres-Mermin inequality

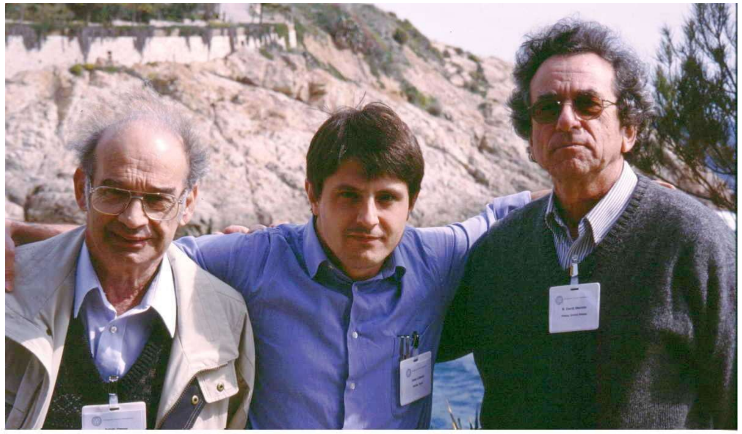
$$\boxed{\langle ABC\rangle + \langle abc\rangle + \langle \alpha\beta\gamma\rangle + \langle Aa\alpha\rangle + \langle Bb\beta\rangle - \langle Cc\gamma\rangle \leq 4}$$

$$A = \sigma_z^{(1)} \qquad B = \sigma_z^{(2)} \qquad C = \sigma_z^{(1)} \otimes \sigma_z^{(2)}$$

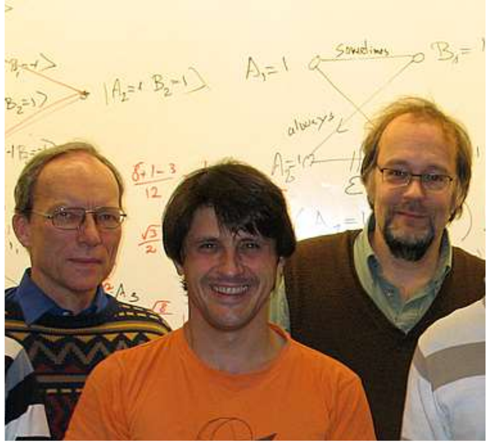
$$a = \sigma_x^{(2)} \qquad b = \sigma_x^{(1)} \qquad c = \sigma_x^{(1)} \otimes \sigma_x^{(2)}$$

$$\alpha = \sigma_z^{(1)} \otimes \sigma_x^{(2)} \qquad \beta = \sigma_x^{(1)} \otimes \sigma_z^{(2)} \qquad \gamma = \sigma_y^{(1)} \otimes \sigma_y^{(2)}$$

- The inequality holds under the assumption of outcome non-contextuality
- Quantum mechanics is not assumed in any way
- The quantum violation occurs for all 4-dim states using the Peres-Mermin observables



2009: Every KS set leads to a SI violation of a NCI



Badziag, P., I. Bengtsson, A. Cabello, and I. Pitowsky (2009), Phys. Rev. Lett. **103**, 050401.

Contextuality for ideal measurements

- It applies to ideal measurements (i.e., those that yield the same outcome when repeated and do not disturb compatible observables)
- It applies to outcome non-contextual models (rather than to non-contextual models satisfying constraints that hold in quantum mechanics)
- It does not require assuming that coarse-grainings of two different measurements represent the same observable. For any observable, the same experimental device can be used in all contexts (as in experiments with sequential measurements)

Problem #2

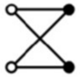
1963: Vorob'yev's theorem

- A violation of noncontextuality inequalities can only occur for scenarios in which the relations of compatibility can be encoded in a graph (in which vertices represent measurements and edges represent relations of mutual compatibility) which is not chordal (i.e., it does not contain induced cycles of size larger than three). Otherwise, there is always a joint probability distribution and, therefore, a non-contextual model

N. N. Vorob'yev, Markov measures and Markov extensions, Theory Probab. Appl. **8**, 420 (1963).
N. N. Vorob'yev, Coalition games, Theory Probab. Appl. **12**, 251 (1967).

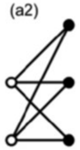
1963: Vorob'yev's theorem

(a1)

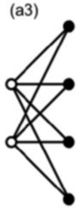


Bohm 1951's
version of EPR
CHSH 1969

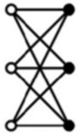
(a2)



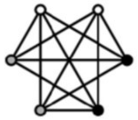
(a3)



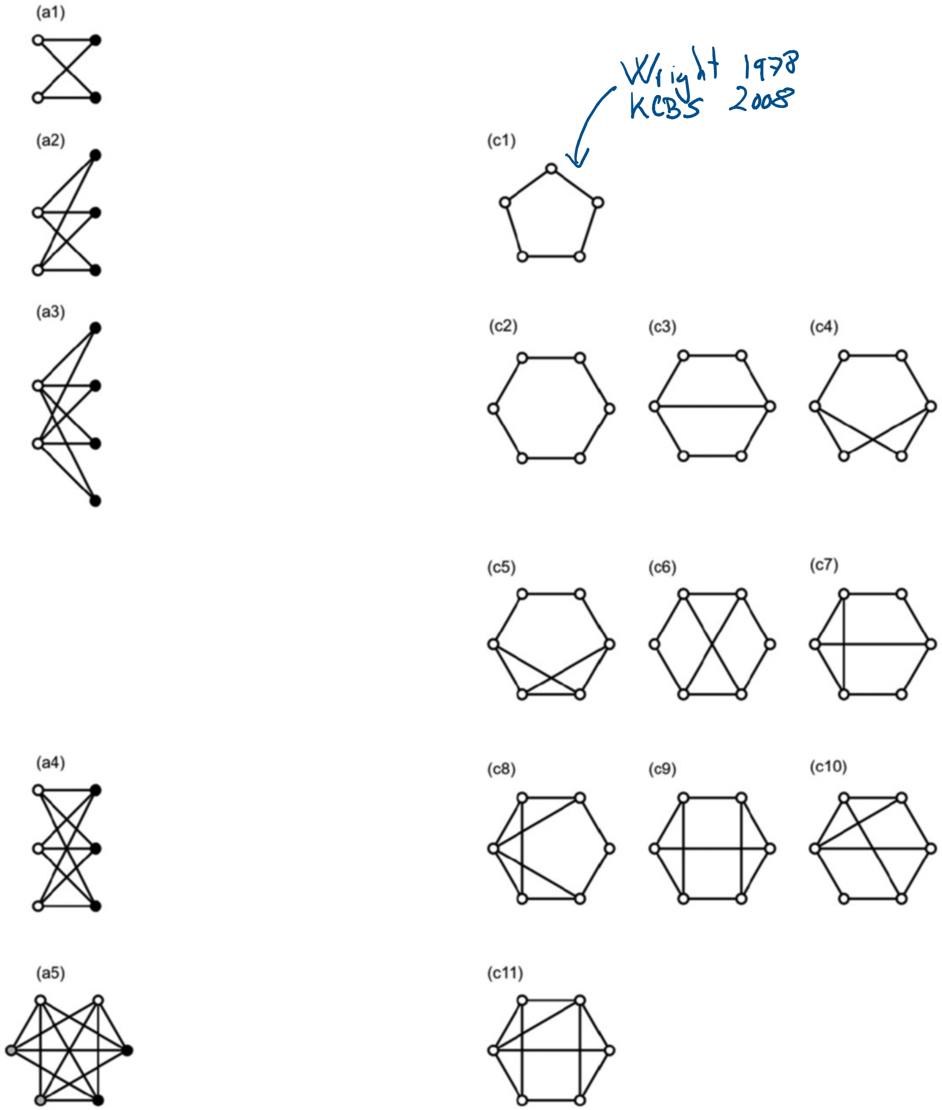
(a4)



(a5)

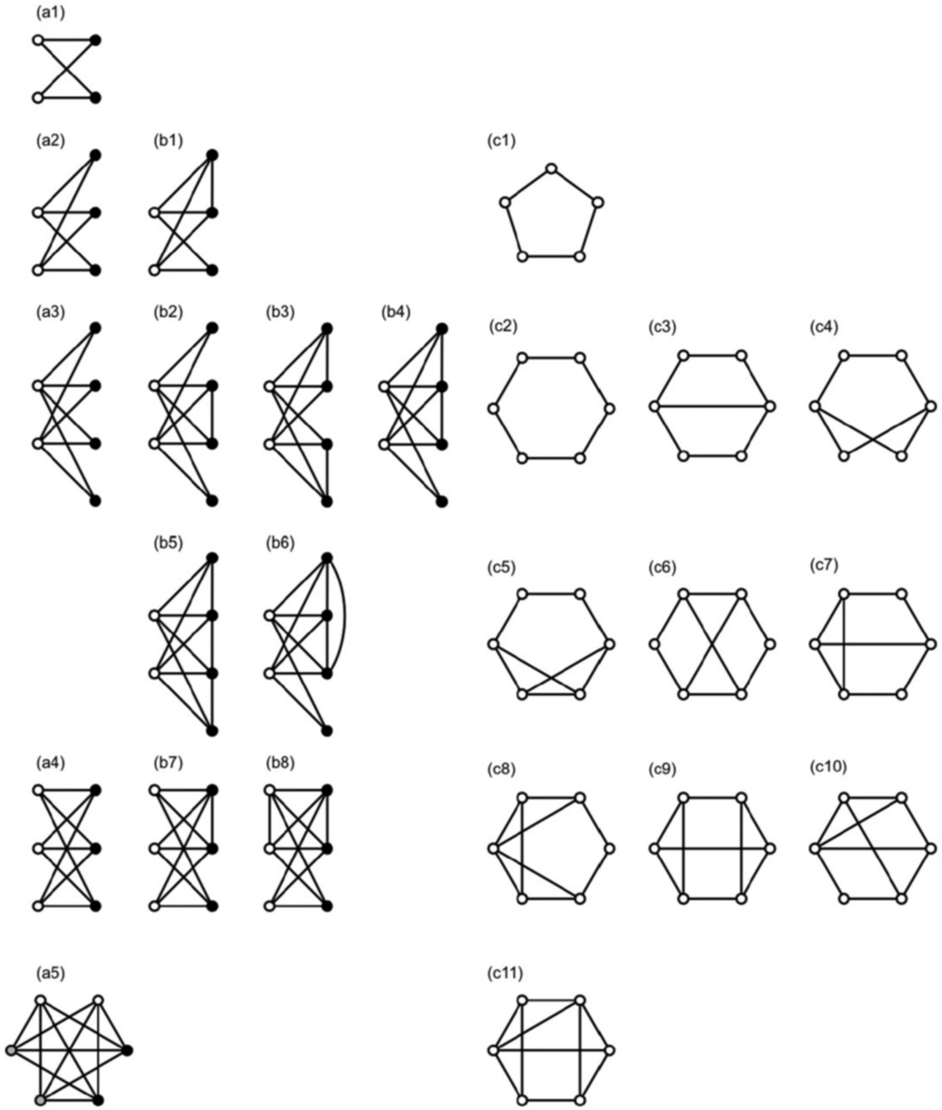


1963: Vorob'yev's theorem



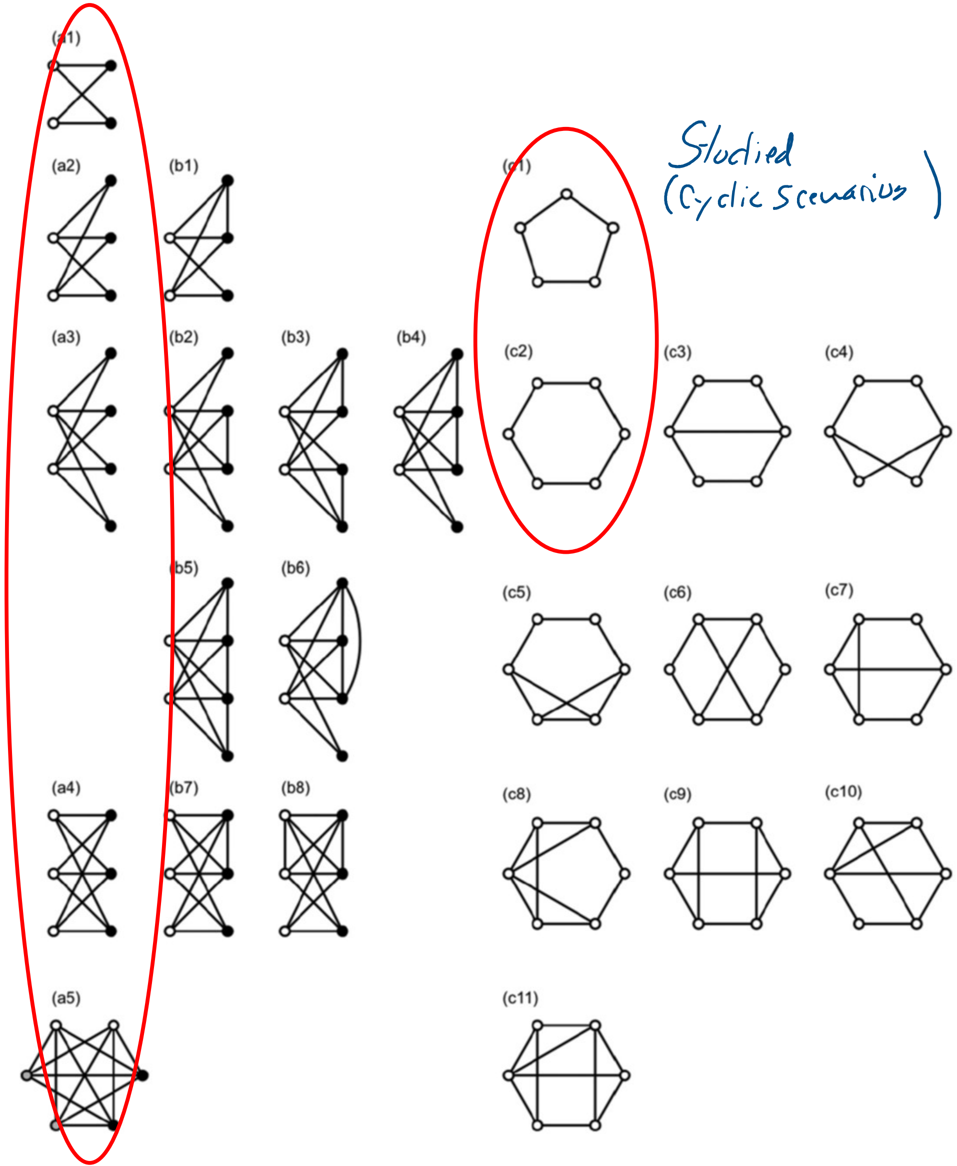
Wright 1978
KCBS 2008

1963: Vorob'yev's theorem



1963: Vorob'yev's theorem

Studied
(Bell scenarios)



Studied
(Cyclic scenarios)

What about
the rest ?

Result #3

The landscape of GPTs

- The framework of Generalized Probabilistic Theories (GPTs) views QT as one possibility in a landscape of theories and asks whether nature could be “more crazy” than quantum
- Some of these theories differ in observable aspects
- E.g., in the set of correlations for Bell scenarios
- E.g., in the set of correlations for scenarios with ideal measurements
- We have not identified a principle that explains the quantum set of correlations for Bell scenarios. Nature could be “more Bell nonlocal” than quantum
- In contrast, nature could not be “more KS contextual”

2019: QT is the most contextual GPT for ideal measurements

- Which noncontextuality inequalities can be violated?
  - Answer: See Vorob'yev's theorem. Quantum theory violates all the inequalities that can be violated
- What is the largest set of correlations for a KS scenario?
  - Answer: The quantum one (assuming that statistically independent copies of any behavior exist and that the theory yields behaviors for any scenario)
- How this compares with quantum theory?
  - Answer: Nature could not be "more KS contextual" than quantum

Cabello, A. (2019b), Phys. Rev. A **100**, 032120.

Problem #3

What does it mean?

- Quantum contextuality is a signature of an *ontological* absence of constraints in the way certain parts of the world interact

Cabello, A. (2019a), Philos. Trans. R. Soc. A **377**, 2019.0136.

- Quantum contextuality simply follows from adopting a Bayesian framework to organize beliefs and update them when new information becomes available

Chiribella, G., A. Cabello, M. Kleinmann, and M. P. Müller (2020), Phys. Rev. Research **2**, 042001.

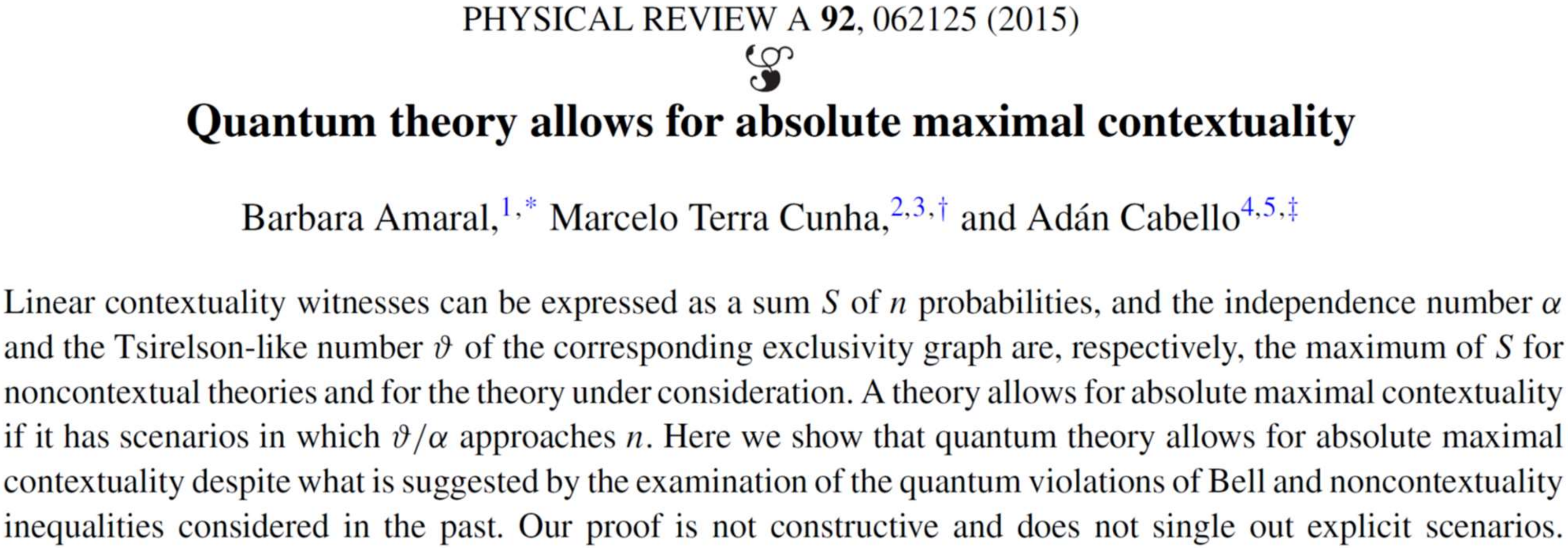
Result #4

2010-2014: The graph-theoretic approach

Cabello, A., S. Severini, and A. Winter (2010), "(non-)contextuality of physical theories as an axiom," ArXiv:1010.2163v1.

Cabello, A., S. Severini, and A. Winter (2014), Phys. Rev. Lett. **112**, 040401.

Nature allows for absolute maximal contextuality



PHYSICAL REVIEW A **92**, 062125 (2015)

Quantum theory allows for absolute maximal contextuality

Barbara Amaral,[1,*] Marcelo Terra Cunha,[2,3,†] and Adán Cabello[4,5,‡]

Linear contextuality witnesses can be expressed as a sum S of n probabilities, and the independence number α and the Tsirelson-like number ϑ of the corresponding exclusivity graph are, respectively, the maximum of S for noncontextual theories and for the theory under consideration. A theory allows for absolute maximal contextuality if it has scenarios in which ϑ/α approaches n. Here we show that quantum theory allows for absolute maximal contextuality despite what is suggested by the examination of the quantum violations of Bell and noncontextuality inequalities considered in the past. Our proof is not constructive and does not single out explicit scenarios.

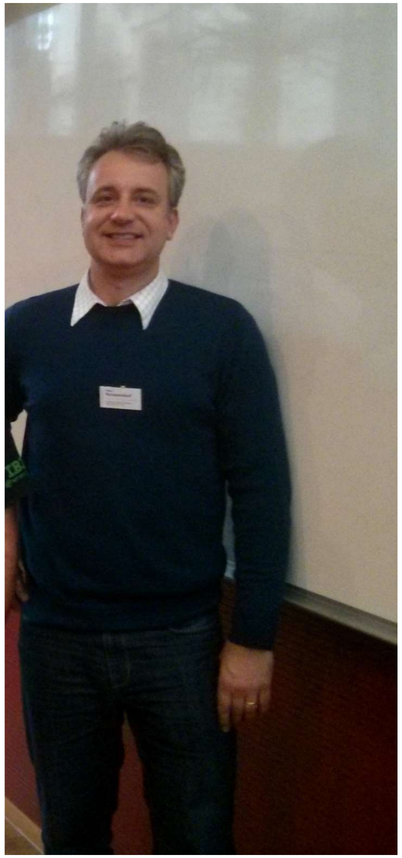
Problem #4

How to produce “absolute maximal contextuality”?

- Or very high quantum contextuality
- Is it robust to noise?
- Is there very high quantum contextuality robust to noise?
- What is it useful for?
- Other measures of contextuality

Result #5

2013: Contextuality in measurement-based q. comp.



Raussendorf, R. (2013b), Phys. Rev. A **88**, 022322.

2014: Contextuality in q. comp. via magic sates

Howard, M., J. Wallman, V. Veitch, and J. Emerson (2014), Nature (London) **510**, 351.

Contextuality in quantum computation. Recommended

- D. Browne, “Contextuality and non-contextuality in (qudit) quantum computing” (video): http://pirsa.org/displayFlash.php?id=17070053

- M. Howard, “Magic sates and contextuality” (slides): https://www.cs.ox.ac.uk/conferences/contextuality/slidesMarkHoward.pdf

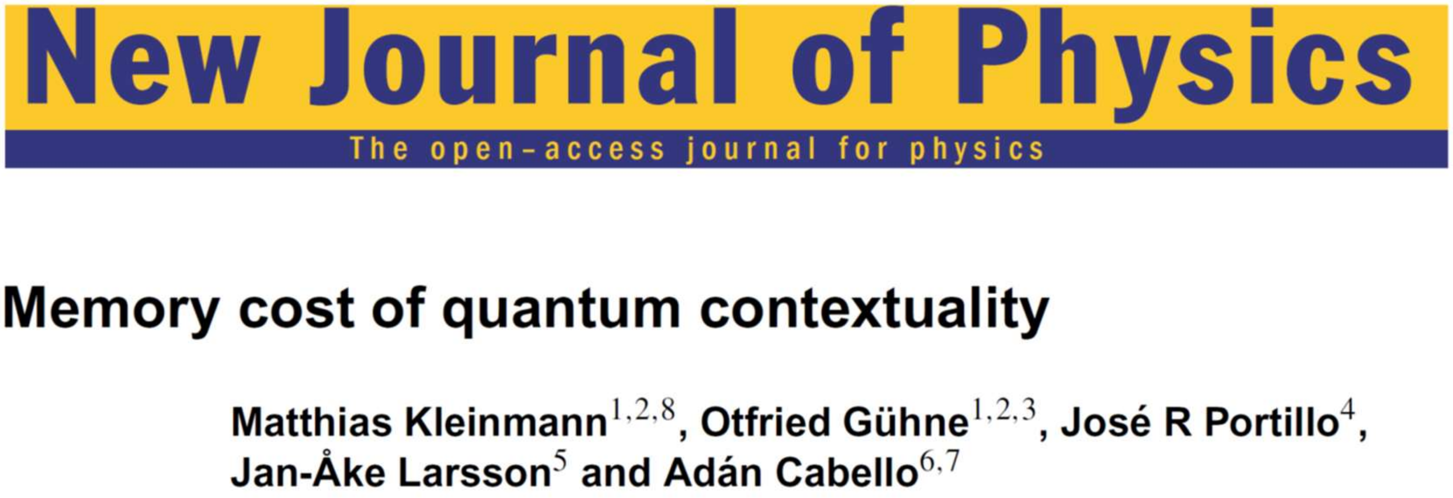
Problem #5

Is contextuality needed in the circuit model?

- More contextuality = more speed up? (Probably no)
- Clarify the relation between contextuality and computation

Result #6

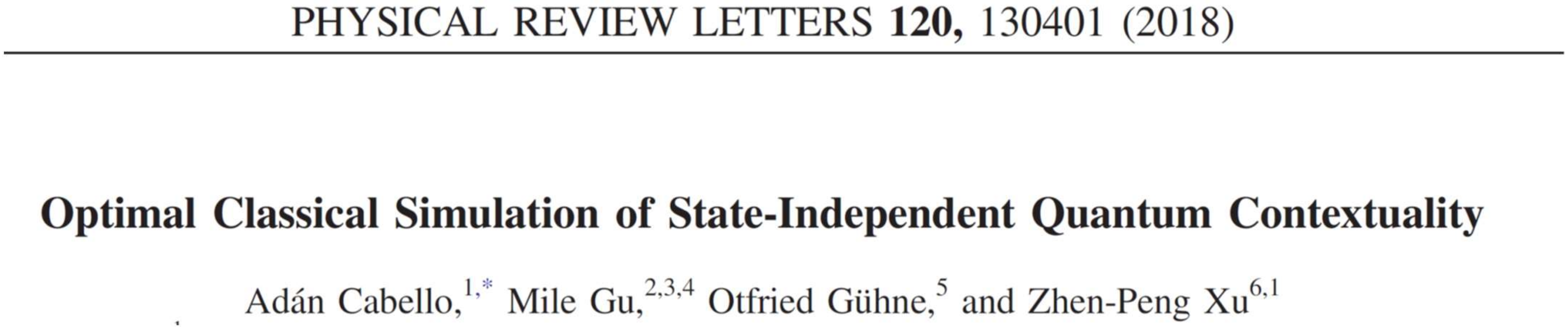
Simulating contextuality requires memory



New Journal of Physics
The open-access journal for physics

Memory cost of quantum contextuality

Matthias Kleinmann[1,2,8], Otfried Gühne[1,2,3], José R Portillo[4], Jan-Åke Larsson[5] and Adán Cabello[6,7]



PHYSICAL REVIEW LETTERS **120,** 130401 (2018)

Optimal Classical Simulation of State-Independent Quantum Contextuality

Adán Cabello,[1,*] Mile Gu,[2,3,4] Otfried Gühne,[5] and Zhen-Peng Xu[6,1]

Problem #6

Computing the memory cost is hard

- Simulating contextuality vs simulating subtheories

- How to compute and maximize the memory cost(s)?

Result #7

Contextuality enables nonlocality

- Every nonlocal correlations are associated to contextual correlations

Problem #7

How to convert every contextuality into nonlocality?

Aim of this talk

- List some open problems

  1. Minimal KS set in dimension 3?
  2. Contextuality in unstudied scenarios
  3. Is contextuality the key property to understand nature?
  4. How to produce “absolute maximal contextuality”?
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Thank you!