Cabello Replies: Marinatto claims [1] that, in the proof of Bell's theorem without inequalities and without alignments [2], local observables cannot be measured by means of tests on individual qubits. We believe that this claim is incorrect. To support this, we explicitly rewrite the proof in terms of tests on individual qubits.

Alice and Bob share eight qubits prepared in the state $|\eta\rangle$ given by Eq. (1) in [2]. Alice has the first four qubits and Bob the remaining four qubits. On her four qubits Alice can measure either $R_A \sigma_{z1}$, $R_A \sigma_{x2}$, $R_A \sigma_{z3}$, and $R_A \sigma_{x4}$, or $R_A \sigma_{z1}$, $R_A \sigma_{x2}$, $R_A \sigma_{z3}$, and $R_A \sigma_{x4}$, where $\sigma_z$ is the spin component of the first qubit along the $z$ direction, and $R_A$ (and $R_B$) is any rotation of Alice’s setups for measuring her four qubits. Analogously, on his four qubits Bob can measure either $R_B \sigma_{x5}$, $R_B \sigma_{x6}$, $R_B \sigma_{z7}$, and $R_B \sigma_{z8}$, or $R_B \sigma_{x5}$, $R_B \sigma_{x6}$, $R_B \sigma_{z7}$, and $R_B \sigma_{z8}$, where $R_B$ (and $R_B$) is any rotation of Bob’s setups for measuring his four qubits. Let us denote by $000\bar{1}, \ldots, 11\bar{1}\bar{1}$, the 16 possible outcomes of Alice (Bob) measuring $R_A \sigma_{z1}$, $R_A \sigma_{x2}$, $R_A \sigma_{z3}$, and $R_A \sigma_{x4}$ ($R_B \sigma_{x5}$, $R_B \sigma_{x6}$, $R_B \sigma_{z7}$, and $R_B \sigma_{z8}$), and by $000\bar{1}, \ldots, 11\bar{1}\bar{1}$, the 16 possible outcomes of Alice (Bob) measuring $R_A \sigma_{z1}$, $R_A \sigma_{z1}$, $R_A \sigma_{x4}$ ($R_B \sigma_{x5}$, $R_B \sigma_{x6}$, $R_B \sigma_{z7}$, and $R_B \sigma_{z8}$). If Alice (Bob) obtains $010\bar{1}, 01\bar{1}0$, $100\bar{1}$, or $101\bar{0}$, she (he) will annotate $F_A = -1$ ($F_B = -1$) as a collective result of her (his) four measurements. In the other 12 possible cases, she (he) will annotate $F_A = 1$ ($F_B = 1$). If Alice (Bob) obtains $0011, 01\bar{1}0$, $1001$, or $1100$, she (he) will annotate $G_A = -1$ ($G_B = -1$) as a collective result of her (his) four measurements. In the other 12 possible cases, she (he) will annotate $G_A = 1$ ($G_B = 1$). With this notation, if Alice measures $R_A \sigma_{z1}$, $R_A \sigma_{z2}$, $R_A \sigma_{x3}$, and $R_A \sigma_{x4}$, and Bob measures $R_B \sigma_{x5}$, $R_B \sigma_{x6}$, $R_B \sigma_{z7}$, and $R_B \sigma_{z8}$, the joint probability that, in the state $|\eta\rangle$, Alice obtains $F_A = 1$ and Bob obtains $F_B = 1$ is

$$P(F_A = 1, F_B = 1) = 0, \quad (1)$$

because, in the state $|\eta\rangle$, the 122 joint probabilities $P(0000, 000\bar{1}), \ldots, P(11\bar{1}\bar{1}, 11\bar{1}\bar{1})$ are zero.

If Alice measures $R_A \sigma_{z1}$, $R_A \sigma_{z2}$, $R_A \sigma_{z3}$, and $R_A \sigma_{x4}$, and Bob measures $R_B \sigma_{x5}$, $R_B \sigma_{x6}$, $R_B \sigma_{z7}$, and $R_B \sigma_{z8}$, the probability that, in the state $|\eta\rangle$, Alice obtains $F_A = 1$, conditioned to Bob obtaining $G_B = 1$ is

$$P(F_A = 1|G_B = 1) = 1, \quad (2)$$

because, in the state $|\eta\rangle$, the 4 × 12 joint probabilities $P(010\bar{1}, 000\bar{1}), \ldots, P(101\bar{0}, 1\bar{1}1\bar{1})$ are zero.

Analogously, if Alice measures $R_A \sigma_{z1}$, $R_A \sigma_{x2}$, $R_A \sigma_{z3}$, and $R_A \sigma_{x4}$, and Bob measures $R_B \sigma_{x5}$, $R_B \sigma_{x6}$, $R_B \sigma_{z7}$, and $R_B \sigma_{z8}$, then in the state $|\eta\rangle$,

$$P(F_B = 1|G_A = 1) = 1, \quad (3)$$

because, in the state $|\eta\rangle$, the 12 × 4 joint probabilities $P(0000, 01\bar{1}0), \ldots, P(11\bar{1}1, 101\bar{0})$ are zero.

Finally, if Alice measures $R_A \sigma_{z1}$, $R_A \sigma_{x2}$, $R_A \sigma_{x3}$, and $R_A \sigma_{x4}$, and Bob measures $R_B \sigma_{x5}$, $R_B \sigma_{x6}$, $R_B \sigma_{x7}$, and $R_B \sigma_{x8}$, then in the state $|\eta\rangle$,

$$P(G_A = 1, G_B = 1) = \frac{9}{112}, \quad (4)$$

because, in the state $|\eta\rangle$, the sum of the 122 joint probabilities $P(0000, 000\bar{1}), \ldots, P(11\bar{1}\bar{1}, 11\bar{1}\bar{1})$ is 9/112.

Equations (1)–(4) allow us to develop a Hardy-like [3] proof of Bell’s theorem without inequalities (see [2] for details), using only single qubit measurements, and with the remarkable property that the setups of Alice and Bob do not need to be aligned, because the required perfect correlations are achieved for any local rotation of the setups.

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