Nonlocality from Local Contextuality

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We experimentally show that nonlocality can be produced from single-particle contextuality by using two-particle correlations which do not violate any Bell inequality by themselves. This demonstrates that nonlocality can come from an \textit{a priori} different simpler phenomenon, and connects contextuality and nonlocality, the two critical resources for, respectively, quantum computation and secure communication. From the perspective of quantum information, our experiment constitutes a proof of principle that quantum systems can be used simultaneously for both quantum computation and secure communication.

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\textbf{Introduction.}—Two famous “no-go” theorems prove that the predictions of quantum theory cannot be explained with hidden variables: Bell’s theorem [1] states that they cannot be reproduced with local hidden variables (LHV) and the Bell-Kochen-Specker (BKS) theorem [2–4] states that they cannot be explained by noncontextual hidden variables (NCHV). Recently, it has been recognized that each of these theorems is behind one of the resources that empower quantum information processing: Bell nonlocality is essential for device-independent secure communication [5–7] and BKS contextuality supplies the power for fault-tolerant universal quantum computation [8–12]. This observation puts the problem of what is the relation between contextuality and nonlocality under a new perspective. In particular, it raises the question of whether single-particle contextuality and two-party nonlocality can coexist, so the same quantum system can provide both resources simultaneously. Surprisingly, the answer to this question is negative if we restrict ourselves to simple forms of nonlocality and single-particle contextuality as, in these cases, there are monogamies between them [13–15] recently observed in experiments [16]. However, Kochen [17], Stairs [18,19], and Heywood and Readhead [20] noticed that the answer is different when single-particle contextuality is state independent. Then, contextuality can be converted into two-particle nonlocality by using Einstein-Podolsky-Rosen (EPR) correlations [21]. It follows that the conflict between quantum theory and LHV theories can be traced back to a conflict between quantum theory and NCHV theories for a single particle. In plain words, nonlocality can be produced from single-particle contextuality by using two-particle correlations which do not violate any Bell inequality by themselves. This connects, in an operational way, a fundamental physical phenomenon, nonlocality, with an \textit{a priori} different phenomenon, single-particle contextuality, providing a new perspective on the origin of nonlocality. Furthermore, from the perspective of quantum information, we see that the two critical resources needed for, respectively, quantum computation and secure communication can both be simultaneously produced by the same system.

\textbf{Experiment.}—In contrast to the standard ways of certifying nonlocality [1,22–26] and single-particle contextuality [27–31], certifying nonlocality originated from single-particle contextuality requires observing the violation of an inequality for LHV theories (i.e., a Bell inequality which follows from the same assumptions under which any Bell inequality is valid), but made of correlations between sequential measurements on one particle and perfect correlations between some of these measurements and the corresponding measurements on a distant particle. The aim of our experiment is testing one of such inequalities proposed in Ref. [32] (see Supplemental Material for a derivation [33]),

\begin{equation}
\langle \omega \rangle \equiv \langle \chi \rangle + \langle S \rangle \leq 16, \tag{1}
\end{equation}

where \(\langle \chi \rangle\) only contains the correlations among the local successive compatible measurements on the first experimenter’s (Alice’s) particle (we will refer to them as Alice-Alice-Alice correlations) and \(\langle S \rangle \) only contains the correlations between the measurements performed by Alice in the second or third place and the measurements performed by the second experimenter, Bob (we will refer to them as Alice-Bob perfect correlations). The fact that \(\langle \omega \rangle\) only contains Alice-Alice-Alice and Alice-Bob perfect correlations is the distinctive signature of inequality (1) with respect to standard Bell inequalities.
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The correlations $\langle \chi \rangle$ and $\langle S \rangle$ are defined as

$$\langle \chi \rangle = \langle CAB \rangle + \langle cba \rangle + \langle \beta \gamma \alpha \rangle + \langle \alpha \lambda \alpha \rangle + \langle \beta \beta \beta \rangle - \langle \gamma \gamma \gamma \rangle,$$

$$\langle S \rangle = |\langle AA' \rangle_CAB| + |\langle BB' \rangle_CAB| + |\langle bb' \rangle_Cba|$$

$$+ |\langle aa' \rangle_Cba| + |\langle \gamma \gamma \rangle_B\rho_B| + |\langle aa' \rangle_B\rho_B| + |\langle bb' \rangle_B\rho_B|$$

$$+ |\langle BB' \rangle_B\rho_B| + |\langle \gamma \gamma \rangle_C\gamma_C| + |\langle CC' \rangle_C\gamma_C|,$$

where $\langle CAB \rangle$ denotes the average of the product of the outcomes of $C$, $A$, and $B$ measured in that order, and $\langle BB' \rangle_{CAB}$ denotes the average $\langle BB' \rangle$ when Alice measures the ordered sequence $CAB$ and Bob measures $B'$. All measurements have two possible outcomes: +1 and −1.

Inequality (1) can be derived from the assumptions of Conway and Kochen’s free will theorem [37,38], but the inequality itself is independent of interpretational issues associated with the theorem, and we use it here to demonstrate that two-particle nonlocality can be produced from another, a priori different, simpler physical phenomenon, i.e., single-particle contextuality.

Our experiment is schematically illustrated in Fig. 1. Two hyperentangled photons (i.e., entangled in two different degrees of freedom) are distributed to two spatially separated laboratories, Alice’s and Bob’s. Alice receives qubits 1 and 2, encoded in, respectively, the spatial mode and the polarization of her photon, and performs three successive measurements on it. Bob receives qubits 3 and 4, encoded in, respectively, the spatial mode and the polarization of his photon, and performs a single measurement on it.

We prepare the two-photon four-qubit state $|\Psi\rangle_{1234} = |\psi^-\rangle_{13} \otimes |\psi^-\rangle_{24}$, where $|\psi^-\rangle_{ij} = (|0\rangle_i \otimes |1\rangle_j - |1\rangle_i \otimes |0\rangle_j)/\sqrt{2}$ is the singlet state for qubits $i$ and $j$. For this purpose, we adopt the scheme shown in Fig. 2. A cw laser at 404 nm pumps two 0.3-mm-thick type-I cut $\beta$-barium borate crystals [39] to generate the two-photon two-qubit entangled state $|\psi^-\rangle = (|H\rangle \otimes |V\rangle - |V\rangle \otimes |H\rangle)/\sqrt{2}$, where $H$ and $V$ correspond to horizontal and vertical polarization, respectively. The experimental concurrence of this state was 0.995 ± 0.003 (statistical errors only). Then, the two photons were sent to the Hong-Ou-Mandel interferometer with visibility 0.996 ± 0.001 (statistical errors only) [40] and then directed to a polarizing beam splitter (PBS). The two photons leave the PBS together through the upper or lower ports and then were split by 50/50 beam splitters (BSs). Next, the spatial modes were
postselected after a careful phase adjustment, so the final state is $|\Psi\rangle_{1234}$. The states $|0\rangle$ and $|1\rangle$ of qubit 1 (qubit 3) were encoded in the “red” paths 1 and 3 (“blue” paths 2 and 4) in Fig. 2. The states $|0\rangle$ and $|1\rangle$ of qubits 2 and 4 were encoded in the $H$ and $V$ polarizations, respectively. We used a coincident count to discard all the events in which both photons are transmitted or reflected by the BSs. A phase stable Sagnac interferometer was adopted to construct the special BS [41] shown in Fig. 2(b). This BS has two advantages: it is polarization independent and its transmission/reflection ratio is controllable and can be set at nearly perfect 50/50.

In our experiment, we had 2 m between Alice’s and Bob’s laboratories and we assumed that this prevents the information about Alice’s (Bob’s) measurement setting from arriving to the photons in Bob’s (Alice’s) laboratory. In addition, we tested that our experimental results are compatible with this assumption by checking that our results do not violate the no signaling principle (see Supplemental Material for details [33]).

The overall detection efficiency was 3.3% and we assumed that the detected photons were a fair sample of the pairs emitted by the source. This assumption can be avoided by adopting high-efficient superconducting detectors and having excellent coupling from the source to the quantum channels and very low loss all the way from the source to the detectors (see Supplemental Material for a discussion of loopholes [33]).

Test of contextuality.—We tested contextuality, by testing the Peres-Mermin inequality [42], which is valid for NCHV theories:

$$\langle \chi \rangle \leq 4.$$  \hspace{1cm} (4)

For this test, Alice measured six sequences: $CAB$, $cba$, $\beta\gamma\alpha$, $\alpha\alpha\alpha$, $\beta\beta\beta$, and $c\gamma C$, where

$$A = \sigma^s_x, \quad B = \sigma^b_p, \quad C = \sigma^c_x \otimes \sigma^c_y,$$

$$a = \sigma^a_x, \quad b = \sigma^b_x, \quad c = \sigma^c_x \otimes \sigma^c_y,$$

$$\alpha = \sigma^a_x \otimes \sigma^a_y, \quad \beta = \sigma^a_x \otimes \sigma^a_y, \quad \gamma = \sigma^c_y \otimes \sigma^c_y, \quad \text{(5)}$$

and $\sigma^s_x$, $\sigma^b_p$, $\sigma^c_x$ denote the Pauli observables corresponding to the spatial mode ($i = s$) and polarization ($i = p$). Each of these nine observables [43,44] was measured using the devices shown in Fig. 3. The configurations corresponding to each of the six sequences are shown in Fig. 4. The designed beam displacer-based interferometer [45] had a visibility of approximately 0.998 using an aligned laser source.

Our experimental result was

$$\langle \chi \rangle = 5.817 \pm 0.011, \quad \text{(6)}$$

which violates the Peres-Mermin inequality (4) by 165 standard deviations. To our knowledge, this is the largest value ever reported for the correlations of the Peres-Mermin inequality [27–29]. Detailed experimental results are provided in the Supplemental Material [33].

Test of perfect correlations.—In the other laboratory, Bob chose among observables $A'$, $B'$, $C'$, $a', b', \alpha'$, and $\gamma'$, which are identical to, respectively, $A, B, C, a, b, \alpha$, and $\gamma$ in Alice’s laboratory (accent marks are just used to remind that these observables are measured on Bob’s photons). For the state $|\Psi\rangle_{1234}$, observables $A$ and $A'$ are perfectly correlated so, by measuring one of them, an experimenter can predict with certainty the result of the corresponding measurement in the distant particle [21]. Similarly for $B$ and $B'$, $C$ and $C'$, $a$ and $a'$, $b$ and $b'$, $\alpha$ and $\alpha'$, and $\gamma$ and $\gamma'$. Consequently, the expected mean value is $\langle S \rangle = 12$ for an ideal experiment. In our experiment we obtained

$$\langle S \rangle = 11.430 \pm 0.016. \quad \text{(7)}$$

Detailed experimental results are provided in the Supplemental Material [33]. The difference with respect to the expected result is due to a nonperfect phase
consideration, then we observed that values of the Alice-Alice-Alice correlations are taken into account. The results of our experiment show that there are correlations revealing Bell nonlocality. From this perspective, the two-particle Bell nonlocality can be produced from testing simpler Bell inequalities \[1,22\]. Therefore, Alice-Bob cooperation represents the setups shown in Fig. 3.

To enable stable sequential measurements, the yellow boxes spatial mode. The specially designed BS [see Fig. 2(b)] was used to recreate the measured eigenstate before entering the next measurement, as our measuring devices map eigenstates to a fixed polarization and entangle them. For that, four identical measuring devices are placed at the outputs of B [28]. Hence, we can determine \((CAB)\) by recording the photon counting probability after measuring device B (from top to bottom named as \(P_1, P_2, \ldots, P_5\), then \((CAB) = P_1 - P_2 + P_3 - P_4 + P_5 - P_6 + P_7 - P_8\). The eigenstates of the measured observable are recreated before entering the next measurement, as our measuring devices map eigenstates to a fixed polarization and spatial mode. The specially designed BS [see Fig. 2(b)] was used to enable stable sequential measurements. The yellow boxes represent the setups shown in Fig. 3.

Compensation in the state preparation. The value of \(\langle S \rangle\) can be reproduced by LHV theories [3,4]. Therefore, Alice-Bob correlations, by themselves, do not reveal nonlocality.

**Test of nonlocality.**—However, when the experimental values of the Alice-Alice-Alice correlations are taken into consideration, then we observed that

\[
\langle \omega \rangle = 17.247 \pm 0.019, \tag{8}
\]

which violates inequality (1) by 66 standard deviations and therefore reveals Bell nonlocality.

**Conclusions.**—Our purpose has been to observe something which cannot be observed in any of the experiments testing simpler Bell inequalities [1,22–26], namely, that two-particle Bell nonlocality can be produced from single-particle contextuality. From this perspective, the results of our experiment show that there are correlations in nature which cannot be explained by LHV theories because they contain single-particle correlations which cannot be reproduced with NCHV theories. This is revealed by the fact that the violation of inequality (1), which proves nonlocality, can be traced back to the violation of inequality (4), which proves single-particle contextuality. It is also revealed by the fact that the correlations between separated particles given by (7), by themselves, admit an explanation in terms of LHV theories, while no such explanation is possible when single-particle correlations are taken into account. Therefore, from this perspective, our experiment shows a new way to produce nonlocality.

In addition, our experiment solves a problem that previous experiments testing the Peres-Mermin inequality [29–27] have. While the results of all these experiments can be simulated with classical models [46–48], our experiment rules out all these models, since no contextual but local hidden variable model can explain the observed correlations. In this sense, our experiment constitutes a crucial development of the experiments in Refs. [27–29].

From the perspective of quantum information, our experiment demonstrates that there is a connection between the two critical resources needed for, respectively, universal fault-tolerant quantum computation and device-independent secure communication. Moreover, our results show that both can be produced simultaneously by the same physical system. This is remarkable in light of recent results proving that this is impossible if we consider simpler forms of contextuality and nonlocality [13–16]. Therefore, our experiment is also a proof of principle that quantum systems can be used simultaneously for quantum computation and secure communication.

Finally, our experiment can also be taken as a test of Conway and Kochen’s free will theorem [37,38]. Under the assumptions in Refs. [37,38], and modulo some loopholes, the violation of inequality (1) implies that the results of the measurements on the photons are not determined by their past.

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[33] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.117.220402 for a derivation of inequality (1), a description of how we tested that there is no signaling, a discussion of some loopholes, and detailed experimental results, which includes Refs. [34–36].
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