“Kochen-Specker vectors” are explicit sets of projectors which prove the Kochen-Specker theorem. The physical content of the Kochen-Specker theorem is that quantum mechanics cannot be simulated by noncontextual hidden-variable theories, or, in Peres’s words, that in quantum mechanics “unperformed experiments have no results”. Its proof is based on the existence of sets of $n$ projection operators, representing yes-no questions, so that none of the $2^n$ possible sets of “yes” or “no” answers is compatible with the sum rule for orthogonal decompositions of the identity (i.e., if the sum of a subset of mutually orthogonal projection operators is the identity, one and only one of the corresponding answers ought to be “yes”). This paper introduces a constructive method for exhaustively generating these sets of projectors. The method, based on three types of algorithms, is then applied to generate all proofs of the Kochen-Specker theorem with less than 25 projectors in dimension 4, and with less than 31 projectors in dimension 3. Remarkably, this approach proves that the set with 18 projectors in dimension 4 discovered by A. Cabello, J. M. Estebaranz and G. García-Alcaine [Phys. Lett. A 212 (1996), no. 4, 183–187; MR1381449 (97a:81016)] is the simplest possible proof of the Kochen-Specker theorem, as conjectured by Peres in 2003.

Reviewed by Adán Cabello

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